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Fermion mass hierarchy and flavour violation in the Froggatt-Nielsen and 331-models

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ACADEMIC DISSERTATION

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Abstract

The Standard Model of particle physics (SM) has been enormously successful in explaining the experimental signals coming from the particle physics experiments. However, it leaves behind some puzzling questions. One of these questions is the *flavour problem.* The SM describes three *generations* of fermions. The everyday world is made of the fermions of the first generation: electron, electron neutrino, up-type quark and the down-type quark. The fermions of the second generation are muon, muon neutrino, charm-quark and strange-quark. The third generation consists of tau lepton, tau neutrino, top quark and the bottom quark. Mathematically each generation is treated identically, so one would expect similar masses for each generation. This is, however, not the case. The first generation is the lightest and the third is the heaviest. For example the top-quark is five orders of magnitude heavier than the up-quark. The SM offers no explanation for this huge span in the fermion masses. This is called the *fermion mass hierarchy problem.*

The fact that the fermions in the SM come in three generations is supported by the experiments. The existence of the fourth generation seems to be excluded. The SM places each generation into identical representation and one could in principle have any number of fermion generations, and still have internally consistent model. Therefore the SM does not answer to the question: why are there exactly three fermion generations in nature? This is called the *fermion family number problem.* The fermion mass hierarchy problem and the fermion family number problem are together known as the flavour problem. In this thesis I concentrate on the possible solutions to the flavour problem. The first part of this thesis contains an overview of the flavour physics in the SM.

The Froggatt-Nielsen mechanism is one of the most popular methods of generating the fermion mass hierarchy. The Froggatt-Nielsen mechanism introduces a new complex scalar field called the flavon and a new global flavour symmetry that forbids the SM Yukawa couplings. When the flavon acquires a non-zero VEV the Yukawa couplings are generated as effective couplings. The hierarchy of the Yukawa couplings and therefore the fermion masses is determined by the charge assignment under this flavour symmetry. The flavon will inevitably have flavour violating couplings and it can mediate processes that are not yet experimentally seen. In the second part of this thesis I discuss the Froggatt-Nielsen mechanism and the flavour violation that it inevitably generates.

The models based on the $SU(3)_c \times SU(3)_L \times U(1)_X$ gauge symmetry are called 331-models. In the traditional 331-models the gauge anomalies only cancel if the number of fermion families is three. The 331-models thus explain the number of fermion generations in nature. The cancellation of gauge anomalies requires that one of the quark generations must be placed into a different representation than the other two. This inevitably leads to the scalar mediated flavour changing neutral currents at tree-level for quarks which are heavily constrained experimentally. This is a problem for the traditional 331-models as they offer no natural suppression mechanism. The traditional 331-models are treated in the third part of this thesis.

Finally in the fourth part of this thesis I discuss the FN331-model which economically incorporates the Froggatt-Nielsen mechanism into the 331-setting. Thus the FN331-model is capable of explaining both the fermion mass hierarchy problem and the fermion family number problem simultaneously, thus solving the flavour problem.

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List of Publications

The publications included in this thesis are

I Higgs-flavon mixing $h \to \mu\tau$,
Katri Huitu, Venus Keus, Niko Koivunen and Oleg Lebedev, **JHEP 1605** (2016) 026, arXiv:1603.06614 [hep-ph].

II Froggatt-Nielsen mechanism in a model with $SU(3)_c \times SU(3)_L \times U(1)_X$ **gauge group**,
Katri Huitu and Niko Koivunen, **Phys. Rev. D98 (2018) no.1, 011701**, arXiv:1706.09463 [hep-ph].

III Suppression of scalar mediated FCNCs in a $SU(3)_c \times SU(3)_L \times U(1)_X$ **-model**,
Katri Huitu and Niko Koivunen, **JHEP 1910** (2019) 065, arXiv:1905.05278 [hep-ph].

The Author's Contribution

I: I did most of the analytical calculations and did the numerical analysis jointly with Dr. Keus. I also participated in the editing of the text.

II: I did the model building jointly with Prof. Huitu. I did all the analytical and numerical calculations, and participated in the editing of the text.

III: I did all the analytical and numerical calculations and wrote the first draft of the text, that was then jointly edited.

Contents

Chapter 1

The Standard Model

1.1 Introduction

The Standard Model of particle physics (SM) has been enormously successful in explaining experimental results. The SM describes the matter and three of the fundamental forces of nature: the strong, the weak and the electromagnetic interaction. The quantum nature of the fourth fundamental force, gravity, is not well understood and it is not usually considered part of the SM.

The matter encountered in everyday life is formed by fermions. Fermions are particles with spin-1/2. The fermions of the SM come in three generations that are also called families. The different generations differ from one another by their masses. The fermions can be further divided into two categories: quarks and leptons. The leptons interact via electroweak force whereas the quarks interact also via strong force in addition to the electroweak one. There are six leptons from which three are electrically charged and three are neutral. The charged leptons are electron e, muon μ and tau τ. They have an electric charge -1. Electron was observed in 1897 [1]. The muon was observed from cosmic rays in 1936 [2], whereas the tau lepton was observed in 1975 [3]. The electrically neutral leptons are called neutrinos. The neutrinos are named after their associated charged leptons. The SM neutrinos are eletron neutrino ν_e, muon neutrino ν_μ and tau neutrino ν_τ. The discovery of the electron neutrino was reported in Ref. [4], muon neutrino in Ref. [5] and tau neutrino in Ref. [6]. There are six quarks, all of which have fractional electric charges. The up-quark u, charm-quark c and top-quark t have electric charge 2/3. The down-quark d, strange-quark s and bottom-quark b have electric charge $-1/3$. The up-, down- and strange-quarks were observed in deep inelastic scattering experiments in 1968 [7],[8], the charm-quark was discovered in 1974 [9],[10], the bottom-quark was discovered in 1977 [11] and finally the top-quark was discovered in 1995 [12], [13].

The strong interaction is responsible for binding quarks into hadrons such as proton and neutron, which form the nucleus of an atom. The strong interaction also binds the protons and neutrons into nucleus. The strong interaction is mediated by massless spin-1 gauge bosons called the *gluons*. The strong interaction is *asymptotically free* [14], meaning that it becomes weaker as the energy is increased. At lower energies the strong interaction becomes *confining*, meaning that it becomes so strong that the quarks cannot exist as free-particles, and are bound into hadrons.

Electromagnetic interaction and gravity are familiar to everyone from everyday life. The electromagnetic interaction is mediated by a massless spin-1 gauge boson called the *photon*. The electromagnetic interaction has infinite range since its mediator is massless. The weak interaction allows for nuclear reactions such as beta decay. The weak force has a very short range as it is mediated by two electrically charged gauge bosons $W_\mu^\pm$ and an electrically neutral gauge boson Z_μ^0, which are massive. The SM unifies the electromagnetic and weak interaction into a single *electroweak* interaction.

In 1933 Fermi formulated the weak interaction as a contact interaction between two charged leptons and two neutrinos [15]. The idea of weak interaction mediated by charged boson was first proposed by Klein [16]. A crucial step towards the modern theory of weak interaction was taken when Feynman, Gell-Mann, Marshak and Sudarshan suggested a model where the weak interaction was mediated by massive vector boson $W_\mu^\pm$ [17]-[18]. This is known as the *Intermediate Vector Boson* (IVB) theory. The IVB theory has a theoretical problem: it is not renormalizable as it contains massive vector bosons as mediators. It can therefore serve only as a phenomenological theory.

In 1961 Glashow proposed a model using $SU(2)_L \times U(1)_Y$ as a symmetry for electromagnetism and weak interactions [19]. This model has the same problem as the IVB theory: the masses of weak gauge bosons had to be added by hand, as the $SU(2)_L \times U(1)_Y$ gauge symmetry forbids the gauge boson mass terms. The model is therefore non-renormalizable. The final step towards the modern theory of electroweak intraction was taken by Weinberg [20] and Salam [21] who introduced spontaneous symmetry breaking [22]-[30] to the Glashow's model to generate the gauge boson masses. The spontaneous symmetry breaking allows to give the gauge bosons masses without breaking the gauge invariance. This Glashow-Weinberg-Salam model is renormalizable as proven by 't Hooft and Veltman [31],[32].

All observed weak interactions until year 1973 were consistent with the hypothesis that all the weak interactions were mediated by $W_\mu^\pm$ bosons only. However, the weak neutral currents were observed at CERN in 1973 [33], [34]. The Glashow-Weiberg-Salam model predicted such interactions and their discovery validated the Glashow-Weinberg-Salam model as the current theory of the electroweak interactions. As the predicted neutral currents were observed in 1973, the search for the electroweak gauge bosons $W_\mu^\pm$ and Z_μ^0 started. The electroweak gauge bosons were observed at CERN in 1983 [35]-[38].

All the particles the SM predicts have been found. The Higgs boson, particle required by the spontaneous symmetry breaking, was the last particle to be found at CERN in 2012 [39],[40].

1.2 Representations

The Standard Model is chiral, that is it treats left- and right-handed fermions differently. The left-handed fermions are placed in the $SU(2)_L$-doublets. The three quark families are embedded into three doublets:

$$Q_{L,1} = \begin{pmatrix} u' \\ d' \end{pmatrix}_L, \quad Q_{L,2} = \begin{pmatrix} c' \\ s' \end{pmatrix}_L, \quad Q_{L,3} = \begin{pmatrix} t' \\ b' \end{pmatrix}_L \sim (3, 2, \frac{1}{3}), \tag{1.1}$$

where we have used the prime in the names of the particles to stress that these fields are gauge eigenstates and not the physical states with a definite mass. The numbers in the parantheses represent the transformation properties of the quarks under the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. The first number in the parantheses tells the transformation property under the strong interaction gauge group $SU(3)_C$. In this case the first number is three, that is left-handed quarks transform under $SU(3)_C$ as a triplet. The second number in the parantheses tells the transformation property under the left-chiral group $SU(2)_L$ and the number two signifies that the left-handed quarks transform as a doublet under $SU(2)_L$. The third number tells the transfomation property under the $U(1)_Y$. The number 1/3 tells that the $U(1)_Y$-charge Y is 1/3. The right-handed quarks do not couple to the $SU(2)_L$ and are placed into $SU(2)_L$-singlets:

$$u'_R,\ c'_R,\ t'_R \sim (3, 1, 4/3), \tag{1.2}$$

$$d'_R,\ s'_R,\ b'_R \sim (3, 1, -2/3). \tag{1.3}$$

The left-handed leptons are also placed into three $SU(2)_L$- doublets:

$$L_{L,1} = \begin{pmatrix} \nu'_e \\ e' \end{pmatrix}_L, \quad L_{L,2} = \begin{pmatrix} \nu'_\mu \\ \mu' \end{pmatrix}_L, \quad L_{L,3} = \begin{pmatrix} \nu'_\tau \\ \tau' \end{pmatrix}_L \sim (1, 2, -1). \tag{1.4}$$

The left-handed leptons do not interact through the strong interaction and transform under $SU(3)_C$ as a singlet. The right-handed charged leptons are placed into $SU(2)_L$-singlets:

$$e'_R,\ \mu'_R,\ \tau'_R \sim (1, 1, -2). \tag{1.5}$$

The SM does not include right-handed neutrinos and, therefore, the neutrinos are left massless in the SM. Note that the SM treats all the fermion generations identically: all the fermion families are placed in same representation.

Fermion	Symbol	Families	$SU(3) \times SU(2)_L \times U(1)_Y$
Quark	$Q_{L,i}$	$\begin{pmatrix} u' \\ d' \end{pmatrix}_L \begin{pmatrix} c' \\ s' \end{pmatrix}_L \begin{pmatrix} t' \\ b' \end{pmatrix}_L$	$(3, 2, \frac{1}{3})$
	$u'_{R,i}$	u'_R, c'_R, t'_R	$(3, 2, \frac{4}{3})$
	$d'_{R,i}$	d'_R, s'_R, b'_R	$(3, 2, -\frac{2}{3})$
Lepton	$L_{L,i}$	$\begin{pmatrix} \nu'_e \\ e' \end{pmatrix}_L \begin{pmatrix} \nu'_\mu \\ \mu' \end{pmatrix}_L \begin{pmatrix} \nu'_\tau \\ \tau' \end{pmatrix}_L$	$(3, 2, -1)$
	$e'_{R,i}$	e'_R, μ'_R, τ'_R	$(3, 2, -2)$

Table 1.1: The fermions of the Standard Model and their transformation properties under the $SU(3)_L \times SU(2)_L \times U(1)_Y$ gauge group.

The gauge interactions are mediated by the spin-1 gauge bosons. The number of gauge bosons associated to each group in $SU(3)_C \times SU(2)_L \times U(1)_Y$ is the number of generators in that group. The $SU(3)_C$ has eight generators and therefore the strong interaction is mediated by eight gluons. The $SU(2)_L$ has three generators and is associated with three W-bosons. The $U(1)_Y$ has only one generator and has one hypercharge gauge boson B.

Particle	Gauge group	$SU(3) \times SU(2)_L \times U(1)_Y$
G^a_μ $(a = 1, ..., 8)$	$SU(3)_C$	$(8, 1, 0)$
W^i_μ $(i = 1, ..., 3)$	$SU(2)_L$	$(1, 3, 0)$
B_μ	$U(1)_Y$	$(1, 1, 0)$

Table 1.2: The gauge bosons of the Standard Model and their transformation properties under the $SU(3)_L \times SU(2)_L \times U(1)_Y$ gauge group.

The Standard Model still contains one more particle, spin-0 scalar, the Higgs field:

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \sim (1, 2, 1). \tag{1.6}$$

The Higgs field plays a crucial role in the SM, by allowing the electroweak gauge bosons and the fermions to acquire a mass that is otherwise forbidden by the gauge symmetry.

1.3 Interactions

The three fundamental interactions are mediated by the gauge bosons. The *gauge principle* uniquely determines the form of the interactions between the matter and the gauge bosons. The fundamental interactions are not, however, enough for a theory to be in agreement with the experiments. One needs to add interactions with a scalar Higgs field, in order to generate masses for the SM gauge bosons and fermions. The Lagrangian density describing the SM can be divided into three parts:

$$\mathcal{L}_{SM} = \mathcal{L}_{gauge} + \mathcal{L}_{fermion} + \mathcal{L}_{Higgs}. \tag{1.7}$$

The first part $\mathcal{L}_{gauge}$ includes the gauge invariant kinetic terms of gauge bosons:

$$\mathcal{L}_{gauge} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} - \frac{1}{4}W^i_{\mu\nu}W^{i\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}, \tag{1.8}$$

where the field strength tensors of the gauge bosons are:

$$G^a_{\mu\nu} = \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_c f_{abc} G^b_\mu G^c_\nu, \tag{1.9}$$

$$W^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g_2 \epsilon_{ijk} W^j_\mu W^k_\nu, \tag{1.10}$$

$$B_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{1.11}$$

The g_c is the $SU(3)_C$ gauge coupling and the g_2 is the $SU(2)_L$ gauge coupling. The f_{abc} and ϵ_{ijk} are the structure constants of $SU(3)_C$ and $SU(2)_L$, respectively. The field strength tensors of the non-Abelian groups have terms proportional to the gauge couplings as seen in Eqs. (1.9) and (1.10). One can therefore easily see that the gauge fields of the non-Abelian groups $SU(3)_C$ and $SU(2)_L$ have self-interactions. The field strength tensor of the Abelian group $U(1)_Y$ does not have a term proportional to the gauge coupling, and it does not exhibit self-interactions.

The second part of the SM Lagrangian density describes the interactions between the SM fermions and the gauge bosons:

$$\begin{aligned}\mathcal{L}_{fermion} &= \sum_{i=1}^{3}\left[i\bar{L}_{L,i}\gamma^\mu D^L_\mu L_{L,i} + i\bar{e}'_{R,i}\gamma^\mu D^e_\mu e'_{R,i}\right] \\ &+ \sum_{i=1}^{3}\left[i\bar{Q}_{L,i}\gamma^\mu D^Q_\mu Q_{L,i} + i\bar{u}'_{R,i}\gamma^\mu D^q_\mu u'_{R,i} + i\bar{d}'_{R,i}\gamma^\mu D^q_\mu d'_{R,i}\right]\end{aligned} \tag{1.12}$$

where the covariant derivatives are:

$$D_\mu^Q = \partial_\mu - i\frac{g_Y}{2}YB_\mu - ig_2\frac{\sigma^i}{2}W_\mu^i - ig_c\frac{\lambda^a}{2}G_\mu^a, \quad (1.13)$$

$$D_\mu^q = \partial_\mu - i\frac{g_Y}{2}YB_\mu - ig_c\frac{\lambda^a}{2}G_\mu^a, \quad (1.14)$$

$$D_\mu^L = \partial_\mu - i\frac{g_Y}{2}YB_\mu - ig_2\frac{\sigma^i}{2}W_\mu^i, \quad (1.15)$$

$$D_\mu^e = \partial_\mu - i\frac{g_Y}{2}YB_\mu, \quad (1.16)$$

where the g_Y is the $U(1)_Y$ gauge coupling, Y is the hypercharge of the fermion that covariant derivative acts on. The $\sigma^i/2$ are the $SU(2)_L$ generators, where the σ^i are the Pauli matrices. The $\lambda^a/2$ are the generators of the $SU(3)_C$, where the $\lambda^a/2$ are the Gell-Mann matrices. The first term in both covariant derivatives is the usual space-time derivative and this will produce the fermion kinetic term in Eq. (1.12). All the fermions couple to the hypercharge gauge boson B_μ (second term in Eqs. (1.13) - (1.14)), but with different strengths due to different hypercharges. The $U(1)_Y$ is therefore chiral. Only the left-handed fermions couple to the $SU(2)_L$ gauge bosons (Eqs (1.13) and (1.15)). Finally only the quarks couple to gluons (Eqs. (1.13) and (1.14)), with both chiralities coupling with the same coupling: $SU(3)_C$ is vector-like.

A model with only gauge interactions will leave fermions and the gauge bosons massless. This is in clear violation with the phenomenological observations. To circumvent this serious problem, Standard Model employs a non-gauge interaction with the scalar field called the Higgs field. The third part of the SM Lagrangian in Eq. (1.7) represents all the interactions with Higgs field.

$$\mathcal{L}_{Higgs} = (D^{\Phi\mu}\Phi)^\dagger(D_\mu^\Phi\Phi) + \mathcal{L}_{Yukawa} - V(\Phi), \quad (1.17)$$

where the covariant derivative acting on the Higgs doublet Φ is:

$$D_\mu^\Phi = \partial_\mu - i\frac{g_Y}{2}YB_\mu - i\frac{g_2}{2}\sigma^i W_\mu^i. \quad (1.18)$$

The first term in Eq. (1.17) gives the gauge interactions of the Higgs doublet. The Higgs doublet couples to hypercharge gauge boson B_μ and the $SU(2)_L$ gauge bosons. The two other terms describe non-gauge interactions. The second term in Eq. (1.17) describes the scalar interacting with fermions, the so-called Yukawa interactions:

$$\mathcal{L}_{Yukawa} = -\sum_{i,j=1}^{3}\left[Y_{ij}^d\bar{Q}_{L,i}\Phi\ d'_{R,j} + Y_{ij}^u\bar{Q}_{L,i}\widetilde{\Phi}\ u'_{R,j} + Y_{ij}^e\bar{L}_{L,i}\Phi\ e'_{R,j}\right] + h.c., \quad (1.19)$$

where $\widetilde{\Phi} = i\sigma_2\Phi^*$ and Y^f, with $f = d, u, e$ are arbitrary 3×3 complex matrices.

The last term in the Eq. (1.17) is the *Higgs potential* that describes the Higgs field interaction with itself. This self interaction is the crucial ingredient for the mass generation in the Standard Model, the so-called *Higgs mechanism*.

1.4 Higgs Mechanism

There are no explicit mass terms for the gauge bosons, as the $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge symmetry prevents one from explicitly writing them. This is in contradiction with the experiments, from which it is well known that the electroweak gauge bosons are massive. This was a setback for the gauge theories, since the gauge principle predicts all the interactions correctly and it would have been unfortunate to discard it. The addition of gauge boson masses by hand violates gauge invariance, and as a result the theory would be non-renormalizable. The SM circumvents this problem of massless gauge bosons by utilizing the Higgs mechanism. The Higgs mechanism will generate the gauge boson masses without violating the gauge invariance, and the theory will remain renormalizable.

1.4.1 Spontaneous symmetry breaking

The spontaneous symmetry breaking takes place when the ground state of the theory is not invariant under the symmetry of the Hamiltonian (or Lagrangian). Let the Hamiltonian, H, be invariant under a symmetry transformation U that is $[H, U] = 0$. Let E_0 be the energy of the ground state,

$$H|0\rangle = E_0|0\rangle. \tag{1.20}$$

If the ground state, $|0\rangle$, is not invariant under U,

$$U|0\rangle \neq |0\rangle, \tag{1.21}$$

the new state $U|0\rangle$ is degenerate to the ground state,

$$H(U|0\rangle) = UH|0\rangle = E_0(U|0\rangle). \tag{1.22}$$

The ground state is related to other degenerate vacua by a continuous symmetry transformation and therefore there must be a continuum of degenerate ground states. In quantum field theory one must pick one of the degenerate vacua to be the ground state around which the particles are excited. This arbitrary choice creates asymmetric ground state. Once the theory is expanded around the asymmetric minimum, the resulting states do no longer exhibit the original symmetry of the theory. The symmetry is now spontaneously broken or maybe more appropriately, *hidden*.

The spontaneous symmetry breaking was first used in particle physics context by Nambu and Jona-Lasinio [22]-[24]. The spontaneous symmetry breaking is a way to introduce masses to the gauge bosons. As stated before the explicit mass terms for the gauge bosons are forbidden by the gauge symmetry itself. Englert and Brout [25], and independently Higgs [26] discovered that the scalar fields with a vacuum expectation value (VEV) give a mass term to the gauge bosons. There was a problem related to the development of models where the gauge boson masses are produced in the spontaneous symmetry breaking, due to Goldstone's theorem [27], [28]. The Goldstone's theorem states that the breaking of continuous symmetry leads to a massless scalar, the *Goldstone boson*, in the physical spectrum. Such particles are however not observed. Guralnik, Hagen and Kibble [29], [30] continued the work and showed that when the gauge bosons acquire masses through the spontaneous symmetry breaking, the would-be-Goldstone bosons are obsorbed into the longitudinal polarization states of the now massive gauge bosons and hence no Goldstone bosons appear in the physical spectrum.

1.4.2 Elecroweak symmetry breaking

In 1961 Glashow proposed $SU(2)_L \times U(1)_Y$ as a symmetry for the electroweak interactions [19]. Later Weinberg [20] and Salam [21] used spontaneous symmetry breaking in a model with $SU(2)_L \times U(1)_Y$ gauge group.

The electroweak gauge bosons, $W^\pm$ and Z^0 are massive. In the Standard Model they acquire masses through the spontaneous symmetry breaking of electroweak gauge symmetry $SU(2)_L \times U(1)_Y$ to electromagnetism:

$$SU(2)_L \times U(1)_Y \to U(1)_{\text{em}}. \tag{1.23}$$

In the electroweak symmetry breaking three electroweak gauge bosons acquire masses, corresponding to three broken symmetry generators. One linear combination of diagonal generators remains unbroken, the electric charge Q:

$$Q = \frac{\sigma_3}{2} + \frac{Y}{2}. \tag{1.24}$$

The Higgs potential of the SM is

$$V(\Phi) = -\mu_h^2 \Phi^\dagger \Phi + \lambda_h (\Phi^\dagger \Phi)^2. \tag{1.25}$$

The coupling λ_h must be positive in order the potential to be bounded from below. The potential then has two different minima depending on the sign of the parameter μ_h^2. When the parameter μ_h^2 is negative, the potential has the minimum at the origin. The minimum is invariant under $SU(2)_L \times U(1)_Y$ transformation and therefore spontaneous symmetry breaking does not take place. When the parameter μ_h^2 is positive the potential has a degenerate minimum at:

$$\Phi^\dagger \Phi = \frac{1}{2}\left(\frac{\mu_h^2}{\lambda_h}\right). \tag{1.26}$$

The ground state is not invariant under gauge transformation and the *spontaneous symmetry breaking* takes place. The vacuum consists of a continuum of states. The non-zero VEV is assigned to the neutral component of the Higgs doublet (Eq. (1.6)), this way the electric charge will not get spontaneously broken.

The three Goldstone bosons, corresponding to the three broken generators, can be removed from the Lagrangian with a proper gauge choice. This gauge choice is called the *unitary gauge*. The Higgs doublet in unitary gauge is:

$$\Phi = \frac{1}{\sqrt{2}}\left(\begin{array}{c} 0 \\ h+v \end{array}\right), \tag{1.27}$$

where h is the Higgs boson and,

$$v = \sqrt{\frac{\mu_h^2}{\lambda_h}}, \tag{1.28}$$

is the vacuum expectation value of the Higgs. When the SM Lagrangian is written in terms of this, the electroweak symmetry is not manifest anymore and becomes hidden. Now the h is a fluctuation around the minimum. The VEV component gives new terms to the Lagrangian including the mass terms for the 3 electroweak gauge bosons and SM fermions.

1.4.3 Mass Generation

Electroweak gauge boson masses

The Lagrangian can now be written in terms of Eq. (1.27). The kinetic term in the Eq. (1.17) will now produce the mass terms for the electroweak gauge bosons at the vacuum:

$$\begin{aligned}\mathcal{L}_{Higgs} &= (D^{\Phi\mu}\Phi)^\dagger (D^\Phi_\mu \Phi) \supset (D^{\Phi\mu}\langle\Phi\rangle)^\dagger (D^\Phi_\mu\langle\Phi\rangle) \\ &= \frac{1}{2}\begin{pmatrix} W_{3\mu} & B_\mu \end{pmatrix}\left(\begin{array}{cc} \frac{1}{4}v^2 g_2^2 & \frac{1}{4}v^2 g_2 g_Y \\ \frac{1}{4}v^2 g_2 g_Y & \frac{1}{4}v^2 g_Y^2 \end{array}\right)\left(\begin{array}{c} W_3^\mu \\ B^\mu \end{array}\right) + \frac{g_2^2}{4}v^2 W_\mu^- W^{+\mu},\end{aligned}$$

where

$$W^{\pm\mu} = \frac{1}{\sqrt{2}}(W_1^\mu \mp i W_2^\mu). \tag{1.29}$$

The first term in Eq. (1.29) shows that the gauge eigenstates are not mass eigenstates. The mass matrix can be diagonalized by a field redefinition:

$$\begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix}, \tag{1.30}$$

where the Z^μ is the Z-boson and A^μ is the photon. The mixing between gauge eigenstates W_3^μ and B^μ is given by the Weinberg angle θ_W defined as:

$$\sin\theta_w = \frac{g_Y}{\sqrt{g_2^2 + g_Y^2}}. \tag{1.31}$$

With the field redefinition Eq. (1.30) the electroweak gauge bosons can be written in the mass eigenstate basis as:

$$(D^{\Phi\mu}\langle\Phi\rangle)^\dagger (D^\Phi_\mu\langle\Phi\rangle) = \frac{1}{2}\begin{pmatrix} Z_\mu & A_\mu \end{pmatrix} \begin{pmatrix} \frac{v^2}{4}(g_2^2 + g_Y^2) & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} Z^\mu \\ A^\mu \end{pmatrix} + \frac{g_2^2}{4} v^2 W^-_\mu W^{+\mu}. \tag{1.32}$$

This shows that the Z-boson and $W^\pm$-boson acquire masses and the photon is massless:

$$m_Z^2 = \frac{v^2}{4}(g_2^2 + g_Y^2), \quad m_W^2 = \frac{v^2}{4} g_2^2 \quad \text{and} \quad \mathrm{m}_\mathrm{A}^2 = 0. \tag{1.33}$$

Before the electroweak symmetry breaking we had four electroweak gauge eigenstate gauge bosons. The electroweak vacuum,

$$\langle\Phi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}, \tag{1.34}$$

breaks three out of four symmetry generators giving mass to three electroweak gauge bosons $W^\pm$ and Z. One linear combination of symmetry generators remains unbroken and corresponding to it one gauge boson remains massless, the photon of the electromagnetism.

Fermion masses

The fermions acquire masses through the Yukawa terms in Eq. (1.19) when the Higgs accquires VEV,

$$\begin{aligned} \mathcal{L}_{Yukawa} \supset & -\sum_{i,j=1}^{3} \left[Y_{ij}^d \bar{Q}_{L,i} \langle\Phi\rangle\ d'_{R,j} + Y_{ij}^u \bar{Q}_{L,i} \langle\widetilde{\Phi}\rangle\ u'_{R,j} + Y_{ij}^e \bar{L}_{L,i} \langle\Phi\rangle\ e'_{R,j} \right] + h.c. \\ = & -\sum_{i,j=1}^{3} \left[m_{ij}^d \bar{d}'_{L,i}\ d'_{R,j} + m_{ij}^u \bar{u}'_{L,i}\ u'_{R,j} + m_{ij}^e \bar{e}'_{L,i}\ e'_{R,j} \right] + h.c., \end{aligned} \tag{1.35}$$

where the fermion mass matrices are

$$m^f = \frac{v}{\sqrt{2}} Y^f, \quad f = d, u, e. \tag{1.36}$$

The fermion mass matrices are in general not diagonal nor Hermitian. Such matrices are diagonalized by a biunitary transformation. The diagonal fermion mass matrix is given by,

$$m^f_{\text{diag}} = \frac{v}{\sqrt{2}} y^f_{\text{diag}} = U^f_L m^f (U^f_R)^\dagger = \frac{v}{\sqrt{2}} U^f_L y^f (U^f_R)^\dagger, \tag{1.37}$$

where U^f_L and U^f_R are unitary matrices. One sees from the Eq. (1.37) that diagonalization of the fermion mass matrix also diagonalizes the Yukawa coupling matrix. This has important implications regarding the flavour phenomenology of the Higgs. The mass eigenstate fermions are related to the gauge eigenstate fermions through:

$$f_L = U^f_L f'_L \quad \text{and} \quad f_R = U^f_R f'_R, \tag{1.38}$$

where $f = u, d, e$ are the mass eigenstate fermions.

Note: in the SM the neutrinos are massless. The SM only has left-handed neutrino and that is not enough for the mass terms. Also the right-handed component is needed for a mass. Therefore the neutrinos have no mass matrix to be diagonalized and the gauge eigenstates are also the mass eigenstates for neutrinos. We still keep the prime in the neutrino field eventhough the neutrino ν'_L is the mass eigenstate. This is because we will later redefine the neutrino field.

The Lagrangian now has to be written in terms of the gauge eigenstates. The fermion diagonalization matrices play an inportant role in the phenomenology of electroweak interactions. Next we look carefylly into the interactions of the physical particles in the electroweak interactions.

1.5 Flavour Violation in the Standard Model

What is flavour? The SM fermions come in three generations, as stated in the introduction. Leptons and quarks of a given generation are said to have the same flavour. This allows one to count the number of certain flavour in a given process: the particles carry one unit of a particular flavour, whereas antiparticles carry a one negative unit of that flavour. Flavour violating processes are such where the number of some flavour is different between the initial and final states. Experimentally it is known that the quark flavour is violated, whereas no flavour violation of charged leptons has been observed. The Table 1.3 presents the stringent bounds on the charged lepton flavour violating interactions.

	Observable	Present limit
1	$\mathrm{BR}(\mu \to eee)$	1.0×10^{-12}
2	$\mathrm{BR}(\tau \to eee)$	3.0×10^{-8}
3	$\mathrm{BR}(\tau \to \mu\mu\mu)$	2.0×10^{-8}
4	$\mathrm{BR}(\tau^- \to \mu^- e^+ e^-)$	1.7×10^{-8}
5	$\mathrm{BR}(\tau^- \to e^- \mu^+ \mu^-)$	2.7×10^{-8}
6	$\mathrm{BR}(\tau^- \to e^+ \mu^- \mu^-)$	1.7×10^{-8}
7	$\mathrm{BR}(\tau^- \to \mu^+ e^- e^-)$	1.5×10^{-8}
8	$\mathrm{BR}(\mu \to e\gamma)$	5.7×10^{-13}
9	$\mathrm{BR}(\tau \to \mu\gamma)$	4.4×10^{-8}
10	$\mathrm{BR}(\tau \to e\gamma)$	3.3×10^{-8}
11	$\mathrm{CR}(\mu\text{-}e, Au)$	7.0×10^{-13}

Table 1.3: Current experimental bounds on the branching ratios of three–body CLFV decays, magnetic transitions and the conversion rate of $\mu \to e$ [41].

It is experimentally known that the quark flavour violating neutral current processes are suppressed significantly compared to the flavour changing charged current interactions and flavour conserving neutral current interactions. The Tables 1.4 - 1.6 present example processes and their braching ratios to illustrate this. The Standard Model explains this by having no flavour changing neutral currents at tree-level: the neutral gauge bosons and the Higgs boson have flavour diagonal interactions and they can't therefore mediate flavour changing interactions. The charged gauge boson $W_\mu^\pm$ has flavour violating couplings and it mediates the flavour violating charged currents at tree-level. The flavour violating neutral current processes are mediated by $W_\mu^\pm$ bosons at loop-level which explains the suppression their rates have compared to the charged current processes.

1.5.1 Absence of tree-level FCNCs

Neutral current processes are those mediated by electrically neutral bosons at tree-level. In the Standard Model they are mediated by Z_μ^0 and Higgs bosons. The neutral bosons have important property in the SM: they do not have flavour changing couplings. Therefore

FCCC
$\text{BR}(K^+ \to \mu^+\nu_\mu) = (63.56 \pm 0.11)\%$
$\text{BR}(B_d^+ \to D^-\mu^+\nu_\mu) = (2.20 \pm 0.1)\%$
$\text{BR}(D^0 \to K^-\mu^+\nu_\mu) = (3.31 \pm 0.13)\%$

Table 1.4: Branching ratios of flavour changing charged current decays for some mesons [41].

FCNC
$\text{BR}(K_L \to \mu^+\mu^-) = (6.84 \pm 0.11) \times 10^{-9}$
$\text{BR}(B_d^0 \to \mu^+\mu^-) = (1.6\pm^{1.6}_{1.4}) \times 10^{-10}$
$\text{BR}(D^0 \to \mu^+\mu^-) < 6.84 \times 10^{-9}$

Table 1.5: Branching ratios of flavour changing neutral current decays for some mesons [41].

Flavour concerving neutral current
$\text{BR}(\Upsilon(1s) \to \mu^+\mu^-) = (2.48 \pm 0.05)\%$
$\text{BR}(J/\psi \to \mu^+\mu^-) = (5.961 \pm 0.033)\%$

Table 1.6: Branching ratios of flavour conserving neutral current decays for some mesons [41].

the SM does not have flavour changing neutral currents at tree-level. The neutral current processes can also be mediated by charged $W_\mu^\pm$ at loop-level. The flavour changing neutral currents are therefore not absent in SM, but are loop suppressed.

Neutral currents of gauge bosons

The kinetic terms of the fermions in Eq. (1.12) produce the fermion gauge interactions. Using the Eqs. (1.29) and (1.30) with the covariant derivatives in Eqs. (1.13) and (1.16) one obtains the gauge boson interactions with fermions:

$$\mathcal{L}_{\text{NC+CC}} = \mathcal{L}_{\text{NC}} + \mathcal{L}_{\text{CC}}. \tag{1.39}$$

The neutral current part is:

$$\mathcal{L}_{\text{NC}} = \sum_f \left\{ \frac{g_2}{\cos\theta_W} \left[g_L^f \bar{f}'_L \gamma^\mu f'_L + g_R^f \bar{f}'_R \gamma^\mu f'_R \right] Z_\mu + eQ(f) \bar{f}' \gamma^\mu f' A_\mu \right\}, \tag{1.40}$$

with,

$$g_L^f = T_3(f_L) - Q(f)\sin^2\theta_W, \tag{1.41}$$
$$g_R^f = -Q(f)\sin^2\theta_W, \tag{1.42}$$

where $T(f)$ and $Q(f)$ are the weak isospin and the electric charge of the fermion f, respectively. So far we have kept the fermions in their gauge eigenstates. Now we, however, want the physical couplings of fermions to neutral gauge bosons Z_μ^0 and A_μ. We write the Z boson couplings to mass eigenstate fermions by using the Eq. (1.38) in Eq. (1.40):

$$\begin{aligned}
\mathcal{L}_{\text{NC,Z}} &= \sum_f \frac{g_2}{\cos\theta_W} \left[g_L^f \bar{f}'_L \gamma^\mu f'_L + g_R^f \bar{f}'_R \gamma^\mu f'_R \right] Z_\mu \qquad (1.43)\\
&= \sum_f \frac{g_2}{\cos\theta_W} \left[g_L^f \left(\bar{f}_L U_L^f \right) \gamma^\mu \left(U_L^{f\dagger} f_L \right) + g_R^f \left(\bar{f}_R U_R^f \right) \gamma^\mu \left(U_R^{f\dagger} f_R \right) \right] Z_\mu \\
&= \sum_f \frac{g_2}{\cos\theta_W} \left[g_L^f \bar{f}_L \gamma^\mu f_L + g_R^f \bar{f}_R \gamma^\mu f_R \right] Z_\mu.
\end{aligned}$$

We see that the fermion diagolization matrices cancel each other. The Z_μ^0 boson does not have flavour violating interactions. The exact cancellation is no accident, but a result of the fermion representations: all the fermion generations are in the same $SU(2)_L \times U(1)_Y$ representations. This is one of the general conditions for the absence of FCNCs at tree-level [42],[43].

The photon coupling to mass eigenstate fermions can be written as:

$$\begin{aligned}
\mathcal{L}_{\text{NC,A}} &= eQ(f) \bar{f}' \gamma^\mu f' A_\mu = eQ(f) \left[\bar{f}'_L \gamma^\mu f'_L + \bar{f}'_R \gamma^\mu f'_R \right] A_\mu \qquad (1.44)\\
&= eQ(f) \left[\left(\bar{f}_L U_L^f \right) \gamma^\mu \left(U_L^{f\dagger} f_L \right) + \left(\bar{f}_R U_R^{f\dagger} \right) \gamma^\mu \left(U_R^{f\dagger} f_R \right) \right] A_\mu \\
&= eQ(f) \bar{f} \gamma^\mu f A_\mu.
\end{aligned}$$

Also the photon has no flavour changing couplings for the same reason as the Z_μ^0 boson: the fermion diagonalization matrices exactly cancel.

Neutral currents of Higgs

The Higgs Yukawa interactions are given by Eq. (1.19). By inserting the Higgs doublet in unitary gauge of Eq. (1.27) to the Eq. (1.19) we obtain the following:

$$\mathcal{L}_{Yukawa} = -\left[\bar{d}'_L Y^d d'_R + \bar{u}'_L Y^u u'_R + \bar{e}'_L Y^e e'_R\right] \frac{1}{\sqrt{2}}(v+h) + h.c., \tag{1.45}$$

By using Eqs. (1.37) and (1.38) we see that in the fermion mass eigenstate basis the Higgs Yukawa couplings are diagonal and there are no FCNCs related to Yukawa interactions in the SM:

$$\mathcal{L}_{Yukawa} = -\left[\bar{d}_L Y^d_{\text{diag}} d_R + \bar{u}_L Y^u_{\text{diag}} u_R + \bar{e}_L Y^e_{\text{diag}} e_R\right] \frac{1}{\sqrt{2}}(v+h) + h.c. \tag{1.46}$$

The fermion mass matrix is proportional to the Yukawa coupling matrix. Therefore the diagonalization of fermion mass matrix also diagonalizes the fermion Yukawa matrix as can be seen in Eq. (1.37). The fact that the fermion Yukawa coupling matrix is proportional to the mass matrix is due to a specific property of the model: each fermion couples only to one scalar field. This ensures that the fermion mass matrix will be proportional to the Yukawa coupling matrix. This is again a general condition for the absence of FCNCs at tree-level [42],[43].

The gluon coupling to the quarks is flavour diagonal for the same reason as photon and Z^0_μ couplings, and thus also the strong interaction conserves flavour.

1.5.2 Charged currents

The charged current part in Eq. (1.39) is given as:

$$\mathcal{L}_{\text{CC}} = \left(\frac{g_2}{\sqrt{2}}\bar{u}'_{L,i}\gamma^\mu d'_{L,i} W^+_\mu + \text{h.c.}\right) + \left(\frac{g_2}{\sqrt{2}}\bar{\nu}'_{L,i}\gamma^\mu e'_{L,i} W^+_\mu + \text{h.c.}\right). \tag{1.47}$$

Let us write the $W^\pm_\mu$ coupling to quarks in terms of mass eigenstates using Eq. (1.38) as

$$\begin{aligned}\mathcal{L}_{\text{CC,quark}} &= \left(\frac{g_2}{\sqrt{2}}\bar{u}'_L\gamma^\mu d'_L W^+_\mu + \text{h.c.}\right) = \left(\frac{g_2}{\sqrt{2}}\bar{u}_L U^u_L \gamma^\mu U^{d\dagger}_L d_L W^+_\mu + \text{h.c.}\right) \\ &= \left(\frac{g_2}{\sqrt{2}}\bar{u}_L\gamma^\mu V_{CKM} d_L W^+_\mu + \text{h.c.}\right),\end{aligned} \tag{1.48}$$

where

$$V_{CKM} = U^u_L U^{d\dagger}_L \tag{1.49}$$

is the Cabibbo-Kobayashi-Maskawa (CKM) matrix [46],[47]. One sees that unlike in the case of neutral currents, the quark diagonalization matrices do not cancel and as a result the CKM-matrix is non-diagonal. This will result in flavour violating couplings for quarks.

The magnitude of CKM matrix elements have been measured and the current bounds to its elements are [41]:

$$|V_{CKM}| = \begin{pmatrix} 0.97420 \pm 0.00021 & 0.2243 \pm 0.0005 & (3.94 \pm 0.36) \times 10^{-3} \\ 0.218 \pm 0.004 & 0.997 \pm 0.017 & (42.2 \pm 0.8) \times 10^{-3} \\ (8.1 \pm 0.5) \times 10^{-3} & (39.4 \pm 2.3) \times 10^{-3} & 1.019 \pm 0.025 \end{pmatrix}. \tag{1.50}$$

The CKM-matrix is a 3×3 unitary matrix and can be parametrized in terms of three mixing angles and one physical phase:

$$V_{CKM} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 s_3 e^{i\delta} \end{pmatrix}, \tag{1.51}$$

where $s_\alpha \equiv \sin\theta_\alpha$, $c_\alpha \equiv \cos\theta_\alpha$, $\alpha = 1, 2, 3$.

The CKM matrix was first proposed by Kobayashi and Maskawa to introduce CP-violation in 1973 [47]. The CP-violation was discovered in decays of neutral kaon in 1964 [48]. The Kobayashi and Maskawa added then yet unobserved third generation to allow for CP-violating phase. In the two family equivalent of the CKM-matrix the CP-violating phase is not present.

Let us now investigate the charged current of leptons. Let us write the leptonic part of the charged current in terms of the mass eigenstates as:

$$\mathcal{L}_{\text{CC,lepton}} = \left(\frac{g_2}{\sqrt{2}} \bar{\nu}'_L \gamma^\mu e'_L W^+_\mu + \text{h.c.} \right) = \left(\frac{g_2}{\sqrt{2}} \bar{\nu}'_L \gamma^\mu U^{e\dagger}_L e_L W^+_\mu + \text{h.c.} \right), \tag{1.52}$$

where the ν'_L is a mass eigenstate, since the neutrinos are massless. The charged lepton diagonalization matrix U^e_L is a general unitary matrix and it would seem that also the lepton flavour is violated by the charged current interaction. This is, however, not so. All the neutrinos are massless, so any linear combination of them is also a mass eigenstate. This allows us to absorb the charged lepton diagonalization matrix U^e_L into the definition of the neutrino fields:

$$\nu_L = U^f_L \nu'_L. \tag{1.53}$$

The leptonic charged current now becomes,

$$\mathcal{L}_{\text{CC,lepton}} = \left(\frac{g_2}{\sqrt{2}} \bar{\nu}_L \gamma^\mu e_L W^+_\mu + \text{h.c.} \right). \tag{1.54}$$

The charged current interaction of leptons is proportional to unit matrix in flavour space and there is no flavour violation in $W_\mu^\pm$ coupling to leptons. This also means that the lepton flavour is absolutely conserved in the SM.

1.5.3 FCNCs at 1-loop

The neutral currents are mediated by $W_\mu^\pm$ bosons at 1-loop order. Neutral meson mixing and decay are important FCNC processes. These processes are loop suppressed, but also additional suppression is present. This additional suppression is due to *Glashow-Iliopoulos-Maiani* (GIM) mechanism. In 1970 Glashow, Iliopoulos and Maiani introduced then yet unobserved charm quark [44]. This allowed to have unitary mixing matrix, U (two flavour version of the CKM matrix), for u, d, s and c quarks. Due to the unitarity condition,

$$U^\dagger U = 1, \tag{1.55}$$

the rate for neutral meson mixing would vanish if the quark masses were identical. The quark masses are, however, different and only the leading constant term cancels in the relevant amplitude. As a result the neutral meson mixing is proportional to quark mass squared differences. Historically the neutral kaon mixing was calculated with u, d and s quarks in IVB theory [45]. This makes the result quadratically divergent and a cut-off Λ had to be tuned to,

$$\Lambda \sim 3 \text{ GeV}, \tag{1.56}$$

to match the experimental results for neutral kaon mixing. The introduction of the charm quark cancels the divergence. The cut-off is then replaced by the quark mass squared difference:

$$|m_u^2 - m_c^2|. \tag{1.57}$$

This places a bound on charm quark mass: $m_c \sim 3$ GeV. The predicted charm quark was soon discovered in 1974.

1.6 Problems of the Standard Model

Even though the SM has been enormously successful in explaining the experimental results, it still has some shortcomings. Some of these shortcomings are:

- **Neutrino masses:** The neutrinos are massless in the SM due to the absence of the right-handed neutrinos. This is in contradiction with the neutrino oscillation experiments which prove that neutrinos have mass [49]-[53]. The neutrino sector of the SM has to be extended in order to explain the neutrino masses. The charged fermions must be Dirac particles, but the neutrinos could also be Majorana particles. The neutrinos are generally expected to be Majorana particles [54], and this could be shown by observing neutrinoless double beta decay [55]. One of the most popular neutrino mass schemes is the high scale seesaw scheme, [56],[57],[58], which assumes the neutrino masses to be related to unification. The neutrino mass can also be a low scale phenomenon. Alternative low scale neutrino mass schemes are rewieved in [59].

- **Dark matter:** The existence of dark matter was discovered when galactic velocities were measured [60]-[61]. The dark matter interacts feebly with the SM particles and only the gravitational interaction between dark matter and ordinary matter has been observed. The dark matter has been searched for in the Direct Detection experiments [62]-[64] and Indirect Detection experiments [65],[66], all of which have been proven null. The Standard Model does not provide a natural dark matter candidate, and lots of SM extensions for DM have been proposed. One of the simplest dark matter models are the scalar extensions of the SM. The real singlet extension has been studied in [67]-[71].

- **Matter-antimatter asymmetry in the universe:** The universe contains much more matter than antimatter. This asymmetry can in principle be undestood if the so called Sakharov conditions are satisfied [72]. One of the Sakharov conditions is the presence of CP-violation. The SM has one source of CP-violation, the phase in the CKM-matrix [46],[47]. It has however been proven that SM does not provide enough CP-violation for the generation of baryon asymmetry [73]. The SM has to be extended in order to obtain more CP-violation. One would be tempted to extend the SM with the fourth generation to have additional CP-violating phases in the CKM-matrix. Unfortunately the fourth generation is excluded experimentally [95]. Another way to acquire CP-violation is to enlarge the scalar sector. The SM does also not provide first order phase transition [74],[75] and therefore the electroweak baryogenesis is not possible in the SM, further motivating the scalar extension of the SM. The electroweak baryogenesis has been studied in scalar extensions of the SM [76]-[79], in 2HDM [80]-[88] and in singlet extension of 2HDM [89],[90],[91]. In addition to baryogenesis the leptogenesis has been proposed as a mechanism for generating the matter-antimatter asymmetry [92].

- **Flavour problem:** The SM treats the fermion families equally by placing them in the same representations, as can be seen from the Eqs. (1.1) - (1.5). The masses and Yukawa couplings of the charged fermions of the Standard Model, span over many orders of magnitude, as can be seen in Table 1.7. This is a problem: one would expect the Yukawa couplings to have the same order of magnitude as they are treated equally. The Standard Model does not offer an explanation why the Yukawa couplings have to be fine-tuned. This is called the *fermion mass hierarchy problem.* The Standard Model has three fermion families. This matches with the experiments, as the fourth generation of fermions is excluded at this point [95]. The Standard Model would retain its internal consistency even if an arbitrary number of families were added. The Standard Model does not present any justification for the existence of exactly three families. This is called the *fermion family number problem.* The fermion mass hierarchy and family number problem are collectively called the *flavour problem.*

Particle	Mass	Yukawa coupling
e	5.110×10^{-4} GeV	2.938×10^{-6}
μ	0.1057 GeV	6.077×10^{-4}
τ	1.7769 GeV	1.022×10^{-2}
u	$(2.16\pm^{0.49}_{0.26}) \times 10^{-3}$ GeV	1.242×10^{-5}
c	1.27 ± 0.02 GeV	7.301×10^{-3}
t	172.9 ± 0.4 GeV	0.994
d	$(4.67\pm^{0.48}_{0.17}) \times 10^{-3}$ GeV	2.685×10^{-5}
s	$(9.3\pm^{1.1}_{0.5}) \times 10^{-2}$ GeV	5.346×10^{-4}
b	$4.18\pm^{0.03}_{0.02}$ GeV	2.403×10^{-2}

Table 1.7: SM fermion masses and Yukawa couplings.

Chapter 2

Froggatt-Nielsen Mechanism

There are many attempts to explain the mass hierarchy of the charged fermions. These include the use of renormalization-group fixed points [96], texture-zeros in the Yukawa matrices at the unification scale [97], [98], dynamics originating from the dark sector [99]-[102] and even seesaw-type mechanism [103]-[104]. There is however one attempt that stands head and shoulders above the rest: the Froggatt-Nielsen (FN) mechanism [105].

2.1 The Froggatt-Nielsen mechanism

The FN mechanism generates the SM Yukawa couplings through effective operarators that are symmetric under the SM gauge group and some new symmetry as well. No fine-tuning of fundamental couplings is required. The Froggatt-Nielsen mechanism extends the Standard Model with a new symmetry. FN-mechanism has also been implemented in many extensions of SM such as supersymmetric models [106], [107], [108], [109], [110] and Randall-Sundrum models [111]. The new symmetry can be a discrete symmetry such as a global Z_N-symmetry, or a continuous global or local (gauged) symmetry. The $U(1)$ symmetry is typically chosen and is henceforth called $U(1)_{FN}$. Also other more complicated continous symmetries have been considered in the literature [112], [113], [114]. The FN mechanism introduces a complex scalar field, Φ, called the *flavon*, which is a singlet under SM gauge group. The FN mechanism also introduces new fermion fields χ_α, where $\alpha = a, b, c...$ labels the different fields. These χ-fields are called *FN-messengers* and they are assumed to be much heavier than the SM particles, with their masses being heavier than $\mathcal{O}(\text{TeV})$. The number of FN messengers depends on the specifics of the FN charge assignment for the SM fermions.

The SM Higgs, the SM fermions, the flavon and the FN-messengers are charged under the FN-symmetry. The purpose of the FN-symmetry is to forbid the SM Yukawa couplings of Higgs to fermions. The FN-symmetry only allows for Yukawa-type interactions where at least one FN-messenger is involved:

$$A_{i\alpha}\bar{\psi}^f_{L,i}H\chi_\alpha + h.c., \quad B_{i\alpha}\bar{f}_{R,i}\chi_\alpha\Phi + h.c. \quad \text{and} \quad C_{\alpha\beta}\bar{\chi}_\alpha\chi_\beta\Phi + h.c., \tag{2.1}$$

where $\psi^f_{L,i}$ is a left-handed SM fermion doublet, $f_{R,j}$ a right-handed SM fermion, H the SM Higgs doublet, and the $A_{i\alpha}$, $B_{i\alpha}$ and $C_{\alpha\beta}$ are dimensionless coupling constants of order-one. This is important concept in setting up the FN mechanism: the fundamental coupling constants of the theory are not to be fine-tuned. This is in contrast to the SM, where the Yukawa couplings are fine-tuned by many orders of magnitude, in order to generate the required fermion masses.

Now the SM Yukawa-terms do not appear at the Lagrangian level. The SM Yukawa-terms must still be generated, in order to match with the Standard Model at low energies. In order to generate the SM Yukawa terms, one can draw diagrams that manifest SM Higgs, two SM fermions and any number of flavons in the external legs. An example of this kind of diagram is presented in the Fig. (2.1). The amount of virtual FN-messengers a diagram of this type has, depends on the FN-charge assignments of the particles in question.

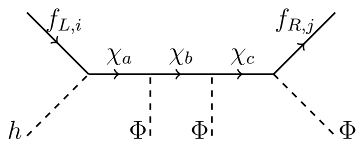

Figure 2.1: An example of a diagram giving rise to the operators responsible to SM Yukawa couplings.

As the FN-messengers are presumably much heavier than the SM particles in the external legs, one can integrate out the virtual FN-messengers. The diagrams of the type presented in Fig. (2.1) then generate the following effective operators:

$$O^{FN}_{ij} = c_{ij}\left(\frac{\Phi}{\Lambda}\right)^{n_{ij}}\bar{\psi}^f_{L,i}Hf_{R,j} + h.c., \tag{2.2}$$

where c_{ij} is a dimensionless complex coupling of order-one and Λ is the mass scale of the FN messengers, assumed to be higher than $\mathcal{O}(\text{TeV})$. The flavon field Φ is decomposed to real and imaginary parts as:

$$\Phi = \frac{1}{\sqrt{2}}(\phi + i\eta), \tag{2.3}$$

and H is the SM Higgs doublet in unitary gauge:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h+v \end{pmatrix}. \tag{2.4}$$

The power n_{ij} is the number of external flavon legs present in the original diagram. This power is determined by the $U(1)_{FN}$-charge conservation. The FN-charges of the relevant fields are given in the Table 2.1.

Particle	Φ	$(\psi^f_{L,i})^c$	$f_{R,i}$	H
FN charge	q_Φ	$q_{\bar{L},i}$	$q_{R,i}$	q_H

Table 2.1: The FN charges of fermions and the scalar fields.

Since the effective operator in Eq. (2.2) is invariant under $U(1)_{FN}$, we obtain:

$$n_{ij} = -\frac{1}{q_\Phi}(q_{\bar{L},i} + q_{f_{R,j}} + q_H). \tag{2.5}$$

2.1.1 Generation of fermion mass hierarchy

Now that we have defined all the incredients we are ready to generate the SM Yukawa couplings through the effective operator in Eq. (2.2). This is accomplished by assuming that the flavon fields accuire a non-zero vacuum expectation value $v_\Phi/\sqrt{2}$. The original operator now becomes:

$$\begin{aligned} O^{FN}_{ij} &\to c_{ij}\left(\frac{\frac{v_\Phi}{\sqrt{2}}+\Phi}{\Lambda}\right)^{n_{ij}} \bar{\psi}^f_{L,i}(H+\langle H\rangle)f_{R,j} \\ &= \underbrace{c_{ij}\left(\frac{v_\Phi}{\sqrt{2}\Lambda}\right)^{n_{ij}}}_{y^f_{ij}} \bar{\psi}^f_{L,i}(H+\langle H\rangle)f_{R,j} \\ &+ n_{ij}\underbrace{c_{ij}\left(\frac{v_\Phi}{\sqrt{2}\Lambda}\right)^{n_{ij}}}_{y^f_{ij}} \frac{\sqrt{2}}{v_\Phi}\bar{\psi}^f_{L,i}(H+\langle H\rangle)f_{R,j}\Phi + ... \\ &= y^f_{ij}\bar{\psi}^f_{L,i}(H+\langle H\rangle)f_{R,j} + n_{ij}y^f_{ij}\frac{\sqrt{2}}{v_\Phi}\bar{\psi}^f_{L,i}\langle H\rangle f_{R,j}\Phi + ... \end{aligned} \tag{2.6}$$

The first term in the last line of Eq. (2.6) corresponds to the SM Yukawa term. The second term in the last line will give rise to flavour violating interactions of flavons with the SM fermions. We return to these flavour violating contributions in the next section. The FN mechanism has a distinct advantage compared to the SM: FN mechanism predicts the hierarchical structure of the Yukawa matrices! To see this, we study the Yukawa matrix generated in the FN mechanism:

$$y_{ij}^f = c_{ij}\left(\frac{v_\Phi}{\sqrt{2}\Lambda}\right)^{n_{ij}}. \tag{2.7}$$

Let us denote quantity in the parantheses as:

$$\epsilon \equiv \frac{v_\phi}{\sqrt{2}\Lambda}. \tag{2.8}$$

If one assumes that $\epsilon < 1$, the order of magnitude of each Yukawa matrix element is determined by the powers of this expansion parameter ϵ. Each Yukawa matrix element is proportional to a complex number c_{ij}, but they are order-one numbers and do not affect the order of magnitude in the elements. The FN-models based on $U(1)$-symmetry are simple, but lack somewhat in their predictibility as there are no relations between the different order-one coefficients of the theory [115], [116], [106]. The non-Abelian groups can produce some relations between the order-one coefficients [112], [113], [114].

Once the value for ϵ is fixed, the choice of FN-charges will determine the powers of ϵ and hence the order of magnitude of the elements. This can be used to acquire naturally the experimentally known eigenvalues of the Yukawa matrix.

The advantage the FN mechanism has is the fact that it gives the fermion mass matrix prior to its diagonalization. This is in contrast to SM where only the singular values of the fermion mass matrices are parametrized. One knows the left- and right-handed fermion diagonalization matrices, presented in Eq. (1.38), now that the fermion mass matrix is known. The CKM-matrix is now calculable once the order-one coefficients of the quark mass matrices are fixed. The order-one coefficients have to be chosen so that that the correct quark masses and CKM-matrix elements are produced. Let us illustrate this by an example for the quark masses.

The expansion parameter ϵ is typically chosen to be the Cabibbo angle: $\epsilon = 0.23$, as this is the natural expansion parameter for the CKM-matrix. The FN-charges of quarks are,

$$\begin{pmatrix} q(Q_{L,1}^c) & q(Q_{L,2}^c) & q(Q_{L,3}^c) \\ q(u_{R,1}) & q(u_{R,2}) & q(u_{R,3}) \\ q(d_{R,1}) & q(d_{R,2}) & q(d_{R,3}) \end{pmatrix} = \begin{pmatrix} 3 & 2 & 0 \\ 5 & 2 & 0 \\ 5 & 3 & 2 \end{pmatrix}. \tag{2.9}$$

With these FN-charges the Yukawa matrix textures of the quarks are:

$$y_u \sim \begin{pmatrix} \epsilon^8 & \epsilon^5 & \epsilon^3 \\ \epsilon^7 & \epsilon^4 & \epsilon^2 \\ \epsilon^5 & \epsilon^2 & \epsilon^0 \end{pmatrix}, \quad \text{and} \quad y_d \sim \begin{pmatrix} \epsilon^8 & \epsilon^6 & \epsilon^5 \\ \epsilon^7 & \epsilon^5 & \epsilon^4 \\ \epsilon^5 & \epsilon^3 & \epsilon^2 \end{pmatrix}. \tag{2.10}$$

The singular values of these matrices are:

$$y_u \sim \epsilon^8, \quad y_c \sim \epsilon^4, \quad y_t \sim \epsilon^0 \tag{2.11}$$

$$y_d \sim \epsilon^8, \quad y_s \sim \epsilon^5, \quad y_b \sim \epsilon^2. \tag{2.12}$$

Their order of magnitude corresponds to those in Table 1.7 and therefore no fine-tuning is required for the coefficients c^q_{ij}. The textures for the left-handed diagonalization matrices are given by,

$$U^L_{ij} \sim \epsilon^{|q(Q_{L,i})-q(Q_{L,j})|}. \tag{2.13}$$

The CKM-matrix texture becomes,

$$V_{CKM} \sim \begin{pmatrix} 1 & \epsilon^1 & \epsilon^3 \\ \epsilon^1 & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix}, \tag{2.14}$$

which corresponds to measured CKM-matrix element values in Eq. (1.50) by order of magnitude.

The FN-mechanism is typically used for the mass generation of the charged fermions. This is due to the fact that the neutrino masses seem to have different origin as they are so much smaller than those of the charged fermions. Nevertheless the FN-mechanism has been applied to the neutrino masses [117], [118], [119]. The flavon Φ remains a physical degree of freedom. The flavon can be light (order of electroweak scale) and its possible collider signatures have been studied [120], [121], [122], [123].

2.2 Possibility for CLFV

The Froggatt-Nielsen mechanism inevitably produces flavour violating effects. Especially interesting is the fact that it can produce flavour violation to the charged lepton sector. In 2015 both CMS and ATLAS collaborations hinted the existence of charged lepton flavour violating decay of the Higgs boson $h \to \mu\tau$ [124], [125]. The combined branching ratio was found to be,

$$\mathrm{BR}(h \to \mu\tau) = 0.82 \pm^{0.33}_{0.32}\ \%. \tag{2.15}$$

This was exteremely interesting, since the charged lepton flavour violation is absolutely forbidden in the SM, as shown in the Section 1.5. The SM therefore predicts this branching ratio to be zero. The BSM physics was needed to explain this signal. A possible BSM scenario providing the required charged lepton flavour violation is provided by the Froggatt-Nielsen mechanism. This approach was taken in [126]. The article studies the large $h \to \mu\tau$ in scenario where the charged lepton flavour violation is due to mixing between the SM Higgs and the flavon. The flavon and the Higgs can mix due to the portal coupling between them [93], [127], [128]. The article concentrates on the *leptofilic* flavon which generates Yukawa sector for the charged leptons only. The quark sector could have its own flavon field to generate its mass hierarchy. The flavour violating contributions would be generated to the quark sector analogously to lepton sector.

The charged lepton couplings to the SM Higgs and the flavon are obtained from the Eq. (2.6):

$$\mathcal{L}_{eff} = m^e_{ij}\overline{e}'_{L,i}e'_{R,j} + \frac{y^e_{ij}}{\sqrt{2}}\overline{e}'_{L,i}e'_{R,j}h + n_{ij}y^e_{ij}\frac{v}{v_\phi}\overline{e}'_{L,i}e'_{R,j}\Phi + ..., \tag{2.16}$$

where the primes in the lepton fields denote gauge eigenstates. This can be presented in the mass eigenstate basis by using the Eq. (1.38):

$$\mathcal{L}_{eff} = \overline{e}_L m^e_{diag} e_R + \overline{e}_L \frac{y^e_{diag}}{\sqrt{2}} e_R h + \overline{e}_L \sqrt{2}\widetilde{\kappa} e_R \Phi + h.c., \tag{2.17}$$

where the flavon coupling is,

$$\widetilde{\kappa} = \frac{1}{\sqrt{2}}\frac{v}{v_\phi}U^e_L(n \cdot y^e)U^{e\dagger}_R \tag{2.18}$$

with

$$(n \cdot y^e)_{ij} = n_{ij}y^e_{ij}. \tag{2.19}$$

The charged lepton mass matrix m^e_{ij} is diagonalized simultaneously with the Yukawa matrix y^e_{ij}. The flavon coupling matrix is however not proportional to the charged lepton Yukawa matrix and is therefore not diagonalized simultaneously with the mass matrix. The physical flavon coupling is thus flavour violating.

The most general scalar potential that respects the $U(1)_{FN}$ symmetry is,

$$V_{FN} = -\mu^2_h(H^\dagger H) - \mu^2_\phi(\Phi^\dagger\Phi) + \lambda_h(H^\dagger H)^2 + \lambda_\phi(\Phi^\dagger\Phi)^2 + \lambda_{h\phi}(H^\dagger H)(\Phi^\dagger\Phi). \tag{2.20}$$

The global $U(1)_{FN}$ symmetry is a continuous symmetry. This symmetry will be spontaneously broken when the flavon acquires a VEV, which will lead to massless Goldstone boson in the physical spectrum according to the Goldstone's theorem [27], [28]. To prevent this the Goldstone boson can be given a mass by explicitly breaking $U(1)_{FN}$ symmetry by a soft mass term,

$$V_{soft} = \widetilde{m}^2\Phi^2 + h.c.. \tag{2.21}$$

The full scalar potential becomes

$$V = V_{FN} + V_{soft}. \tag{2.22}$$

Both the SM Higgs and the flavon acquire a non-zero VEV and the Higgs and the flavon mix due to the portal coupling in the potential. All the parameters in the potential can be made real by a phase redefinition and therefore the SM Higgs will mix only with the real part of the flavon. The CP-even mass eigenstates are:

$$\begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} h \\ \phi \end{pmatrix}. \tag{2.23}$$

The physical Yukawa couplings of scalars and the charged leptons become:

$$\begin{aligned} \mathcal{L} \supset & \left[\cos\theta \frac{y^e_{diag,ij}}{\sqrt{2}} + \sin\theta\widetilde{\kappa}_{ij}\right] \bar{e}_i P_R e_j H_1 + \left[-\sin\theta \frac{y^e_{diag,ij}}{\sqrt{2}} + \cos\theta\widetilde{\kappa}_{ij}\right] \bar{e}_i P_R e_j H_2 \\ + & \; i\widetilde{\kappa}_{ij}\bar{e}P_R e_j \eta + h.c. \end{aligned} \tag{2.24}$$

The first term in the square brackets is the physical Yukawa coupling of 125 GeV Higgs. It contains the diagonal SM contribution, which is however suppressed by $\cos\theta$. The mixing with the flavon has also produced flavour violating contribution $\sin\theta\widetilde{\kappa}_{ij}$. The flavour violating coupling is suppressed by the flavon VEV and the sine of the mixing angle. The flavour violating part can be large for relatively small flavon VEV and large mixing angle.

2.2.1 FN charges and the CLFV bounds

The SM Higgs decays to tau's 6% of the time. The LHC signal in Eq. (2.15) is roughly one order of magnitude smaller than the Higgs decay to tau's. The $\mu\tau$-coupling of H_1 is suppressed by the sine of mixing angle however. The upper bound for the mixing angle is,

$$|\sin\theta| < 0.33, \tag{2.25}$$

according to [129]. This suppresses the $\mu\tau$-coupling by an order of magnitude. The $\widetilde{\kappa}_{\mu\tau}$ and/or $\widetilde{\kappa}_{\tau\mu}$ coupling has to therefore be comparable or even larger than the tau Yukawa coupling of the SM in order to explain the LHC signal in Eq. (2.15).

The mixing angle has to be as large as possible and the flavon VEV has to be roughly same order as the Higgs VEV in order to obtain that large $\mu\tau$ coupling. This will put serious strain to the CLFV processes that bound that coupling. The $\widetilde{\kappa}_{\mu\tau}$ and/or $\widetilde{\kappa}_{\tau\mu}$ coupling has to be as large as possible, while keeping the other off-diagonal couplings as small as possible. This can be affected by choosing FN-charges properly. The chosen FN-charges are presented in the Table 2.2. The resulting Yukawa matrix is far from diagonal, which leads to large mixing in $\mu\tau$-sector. The order one coefficients are chosen so that the masses of the charged leptons are produced correctly:

$$y^e = \begin{pmatrix} 3.3855\ \epsilon^6 & -0.625\ \epsilon^6 & 3.5\ \epsilon^7 \\ 5.36\ \epsilon^4 & 6.1465\ \epsilon^4 & -3.125\ \epsilon^5 \\ 0.5\ \epsilon^2 & 0.5\ \epsilon^2 & 7.3312\ \epsilon^3 \end{pmatrix}. \tag{2.26}$$

Particle	$L^c_{L,i}$	$e_{R,j}$
e	6	0
μ	4	0
τ	2	1

Table 2.2: FN charge assignment.

The Yukawa matrix in Eq. (2.26) correspond to the following $\widetilde{\kappa}$-matrix,

$$\widetilde{\kappa} = \frac{v}{v_\phi} \begin{pmatrix} 1 \times 10^{-5} & -1 \times 10^{-6} & -3 \times 10^{-6} \\ -2 \times 10^{-5} & 2 \times 10^{-3} & 6 \times 10^{-4} \\ 3 \times 10^{-4} & -4 \times 10^{-3} & 2 \times 10^{-2} \end{pmatrix}, \tag{2.27}$$

where the flavon VEV is a free parameter. The flavon VEV has to be around the electroweak VEV so that the $\mu\tau$-coupling of H_1 is comparable to the SM tau Yukawa coupling. The explanation of the $h \to \mu\tau$ signal requires a large $\mu\tau$-coupling. The large $\mu\tau$-coupling will potentially make the rates for the CLFV processes $l_i \to l_j l_k l_l$, $\mu \leftrightarrow e$-conversion and $l_i \to l_j \gamma$ large. These processes are tightly constrained as seen in the Table 1.3. All the physical scalars, H_1, H_2 and η can mediate these CLFV processes as presented in the Feynman diagrams in Figures 2.2- 2.4.

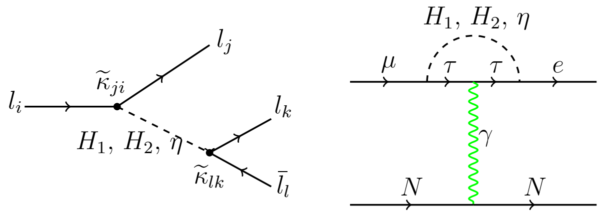

Figure 2.2: The $l_i \to l_j l_k l_l$ (left) and $\mu \leftrightarrow e$-conversion (right) processes mediated by the mass eigenstate scalars H_1, H_2 and η.

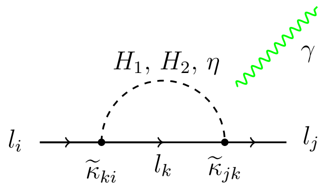

Figure 2.3: The 1-loop contribution to $l_i \to l_j \gamma$ processes mediated by the mass eigenstate scalars H_1, H_2 and η.

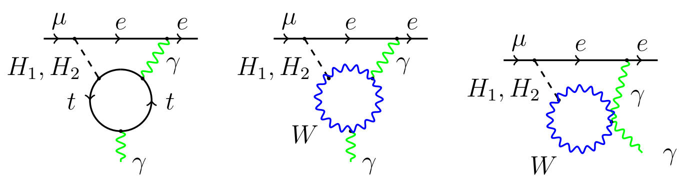

Figure 2.4: 2-loop contribution to $\mu \to e\gamma$.

For the numerical example in Eqs. (2.26) and (2.27) the $\mu \to e\gamma$ is the most constraining CLFV process. Bounds on the mixing angle and the VEV are presented in Figure 2.5. In the narrow portions of the coloured sections are where cancellation between the CP-even and the CP-odd scalars take place. This will make the rate smaller and allow for large mixing angles and small Flavon VEVs. One can see that one can obtain large mixing angles, $\sin\theta \sim 0.3$, and small flavon VEVs, $v_\phi \sim v$, without violating the CLFV bounds. This allows to obtain large rates for $h \to \mu\tau$.

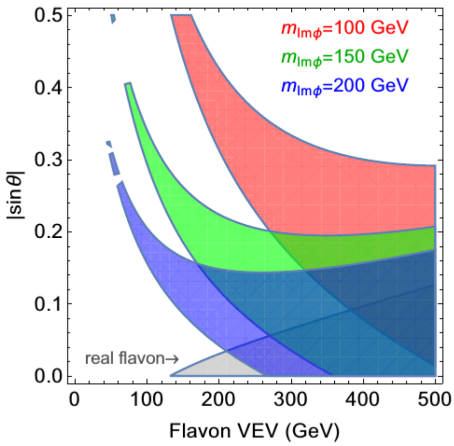

Figure 2.5: The coloured portions are allowed by the LFV constraints for $m_\eta = 100,\ 150,\ 200$ GeV. The second Higgs mass has been set to $m_{H_2} = 500$ GeV.

At tree-level the $H_1 \to \mu\tau$ and $H_1 \to \tau\tau$ decay rates are,

$$\Gamma(H_1 \to \mu\tau) = \frac{m_{H_1}}{8\pi} \sin^2\theta(|\widetilde{\kappa}_{\mu\tau}|^2 + |\widetilde{\kappa}_{\tau\mu}|^2), \tag{2.28}$$

and,

$$\Gamma(H_1 \to \tau\tau) = \frac{m_{H_1}}{8\pi} \left[\cos\theta \frac{y^e_{diag,\tau}}{\sqrt{2}} + \sin\theta \widetilde{\kappa}_{\tau\tau}\right]^2. \tag{2.29}$$

The $H_1 \to \mu\mu$ decay rate is defined analogously to $H_1 \to \tau\tau$. The large $\mu\tau$-coupling of Higgs will make also the $\tau\tau$ and $\mu\mu$ coupling large as flavon VEV close to Higgs VEV is required. The LHC searches, however, pose constraints on the H_1 Yukawa couplings to muon and tau [130], [131], [132].

The LHC bounds on the leptonic Higgs decays assume that the Higgs production cross section is not modified by the new physics. This has to be taken into account. The Higgs production cross section in current model is suppressed by the factor $\cos^2\theta$ compared to the SM. Also the total decay width of the Higgs is suppressed by this factor.

It is convenient to translate the bounds [130], [131], [132] to the effective branching ratio $\mathrm{BR}_{\mathrm{eff}}(H_1 \to l_i l_j)$:

$$\sigma(H_1)\mathrm{BR}(H_1 \to l_i l_j) = \sigma(h)\frac{\Gamma(H_1 \to l_i l_j)}{\Gamma^{\mathrm{total}}_{\mathrm{SM}}(h)} \equiv \sigma(h)\mathrm{BR}_{\mathrm{eff}}(H_1 \to l_i l_j), \tag{2.30}$$

where the $\sigma(h)$ is the SM Higgs production cross section and $\Gamma^{\mathrm{total}}_{\mathrm{SM}}(h) = 4.1$ MeV is the SM Higgs total decay width. The Figure 2.6 presents the $\mathrm{BR}_{\mathrm{eff}}(H_1 \to \tau\tau)$ with the LHC bounds. One can see that the LHC signal can be satisfied simultaneously with the LHC bounds for the Yukawas.

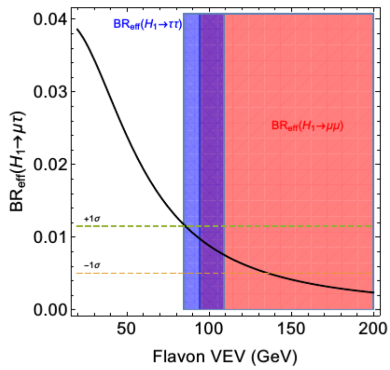

Figure 2.6: The black curve is $\mathrm{BR}_{\mathrm{eff}}(H_1 \to \mu\tau)$ as a function of flavon VEV v_ϕ with $\sin\theta = -0.3$. The red region is allowed by $\mathrm{BR}_{\mathrm{eff}}(H_1 \to \mu\mu)$ at 95% CL and the blue region is allowed by $\mathrm{BR}_{\mathrm{eff}}(H_1 \to \tau\tau)$ and the purple region is allowed by both [130, 131, 132]. The dashed lines show the $\pm 1\sigma$ limits of Eq. (2.15).

The LHC signal from 2015 presented in Eq. (2.15) has since gone away. The latest searches for lepton flavour violating decays of Higgs of the ATLAS and CMS have seen no signal [133], [134]. The work done in [126] is still relevant as it shows that the FN-mechanism is capable of producing large charged lepton flavour violating branching rations of Higgs while still avoiding stringent CLFV bounds. If CLFV processes are ever observed the FN-mechanism is a good candidate for source of CLFV. The observation of charged lepton flavour violation would be a direct proof of the existence of BSM physics.

Chapter 3

331-Models

Models based on $SU(3)_c \times SU(3)_L \times U(1)_X$ gauge group are called 331-models. The 331-models have been advocated to explain the number of fermion families in nature. The 331-models were initially studied as an alternative way to explain the suppression of the FCNC without the GIM mechanism [135]-[151]. Later the 331-models shifted towards explaining the family structure, which we call traditional 331-models [152]-[163], [164]-[175].

The $SU(3)_c \times SU(3)_L \times U(1)_X$ is larger group than the SM gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$. The $SU(3)_L$ has five more generators compared to $SU(2)_L$, and therefore 331-models contain five new gauge bosons. The five additional gauge bosons need to acquire masses in spontaneous symmetry breaking. The scalar sector has to be extended in order to have more would-be-Goldstone bosons for the absorbtion by the new gauge bosons. In 331-models the fundamental representation of fermions is a $SU(3)_L$ triplet. This means that the fermion sector has to be extended as well. The particle content of the 331-models is extended in all sectors. This provides an opportunity in the 331-setting to study many problems left behind by the SM, including: CP-violation [160], strong CP-problem [176], [177], neutrino masses [178], [179], [180], [181], [182], [183], [184], [185]-[188] and dark matter [189]-[197], [198]. The phenomenology of 331-models has been studied in [199],[200]. Also supersymmetric versions of the 331-models have been studied [172],[201].

The most important difference between the SM and the 331-models is the cancellation of chiral anomalies [202]-[204]. Theories, where the gauge boson coupling to fermions depend on their handedness are chiral. The Standard Model is chiral as was seen in Chapter 1. The chiral theories potentially suffer from anomalies related to loop corrections to three-gauge-boson vertex functions. These anomalies would violate gauge invariance and nullify the theory as quantum theory. Theories, where gauge bosons couple to chiral currents, can only be gauge invariant, if anomalous contributions cancel. Gauge theories that are free from anomalies are discussed in [205].

In the 331-models the pure $SU(2)_L$-anomaly of the SM is replaced with the pure $SU(3)_L$-anomaly. In the SM the pure $SU(2)_L$-anomaly cancels due to special property of the $SU(2)_L$-generators:

$$\left\{\frac{\sigma^a}{2}, \frac{\sigma^b}{2}\right\} = \frac{1}{2}\delta^{ab}, \tag{3.1}$$

regardless of the particle content. The $SU(3)_L$-generators do not share this property and the pure $SU(3)_L$-anomaly does not cancel automatically. The fermions have to be arranged into $SU(3)_L$-multiplets in a certain way, in order to cancel the pure $SU(3)_L$-anomaly. The pure $SU(3)_L$-anomaly cancels only if there are equal number of fermion triplets and antitriplets. This is additional constraint from the anomaly cancellation compared to the SM. In the traditional 331-models this is used to explain the number of families [152]-[163], [164]-[175]. This is based on the following assumptions:

- There is only one $SU(3)_L$-multiplet for each generation.
- QCD remains asymptotically free.

The first assumption restricts the number of new fermions. Each fermion triplet and antitriplet will as a result have at most one new fermion. The first assumption alone won't predict three families, but that the number of families is an integer multiple of 3 (number of colors): 3, 6, 9.... However, if the number of families is larger than 4, the theory has too many coloured fermions and the QCD won't be asymptotically free, in contradiction to the observations. Hence the second condition. The only remaining possibility is to have 3 families. Models based on $SU(3)_c \times SU(4)_L \times U(1)_X$ gauge symmetry have also been studied in the literature [156],[206]-[212]. They also predict the number of familes to be three.

The cancellation of chiral anomalies is the celebrated property of the 331-models, but at the same time it produces their worst property. The cancellation of chiral anomalies forces to place one of the quark families into different representations than the other two. This has devastating consequences. The quarks will couple to multiple scalar multiplets, which inevitably leads to flavour changing neutral currents at tree-level [42],[43]. This catastrophe was so succesfully prevented in the SM by treating all the generations equally. The flavour changing neutral currents on the gauge boson sector have been studied, and GIM mechanism is found to work for the Z_μ-boson [154], [159]. The Z'_μ-boson will have flavour changing couplings in general, but their effect is suppressed due to heavy mass of the Z'_μ-boson. The true problem is not with the gauge boson mediation, but with the scalar sector. The plague of tree-level scalar mediated FCNCs of quarks is a general property of traditional 331-models [152]-[163], [164]-[175]. There is no natural suppression mechanism for the scalar mediated FCNCs of quarks in 331-models. In the literature this has been until recently ignored. The literature typically adresses the problem of scalar mediated quark FCNCs by just assuming that the Yukawa coupling structure is such that the FCNCs are suppressed [152], [153], [154], [158], [159], [164], [171]. Recently the suppression of the scalar mediated FCNCs was studied in great detail in a 331-model where the Froggatt-Nielsen mechanism is incorporated [213], [214]. This suppression by Froggatt-Nielsen mechanism is discussed in next Chapter. The lepton generations are treated identically in the traditional 331-models and they do not suffer from any flavour changing effects. It is only the quark sector that suffers from flavour changing effects.

The spontaneous symmetry breaking of $SU(3)_L \times U(1)_X$ takes place at two stages:

$$SU(3)_L \times U(1)_X \overset{\Lambda_{331}}{\rightarrow} SU(2)_L \times U(1)_Y \overset{\Lambda_{EW}}{\rightarrow} U(1)_{em}. \tag{3.2}$$

The first step breaks the 331-model into SM and this takes place at some high energy scale $\Lambda_{331} \gtrsim \mathcal{O}(\mathrm{TeV})$. The electroweak symmetry breaking takes place at electroweak scale Λ_{EW} as usual. The masses of the new gauge bosons are proportional to the $SU(3)_L \times U(1)_X$-breaking VEV and they are very heavy.

The $SU(3)_L \times U(1)_X$ gauge group contains one additional diagonal generator compared to the SM. This gives freedom in the way the electric charge is embedded into the $SU(3)_L \times U(1)_X$. The electric charge can be writen in a general form as:

$$Q = T_3 + \beta T_8 + X, \tag{3.3}$$

where T_3 and T_8 are the diagonal generators of $SU(3)_L$ and X is the $U(1)_X$-charge. The real parameter β defines the type of the 331-model. The models with $\beta = \pm 1/\sqrt{3}$ [152]-[163] and $\beta = \pm\sqrt{3}$ [164]-[175] are well studied in literature. Also models with $\beta = 0$ have been studied [215],[216]. The models with different β differ in their particle content. The models with $\beta = \pm\sqrt{3}$ contain particles with exotic electric charges, such as quarks with electric charges $\pm 4/3$ and $\pm 5/3$, and doubly charged scalars and gauge bosons. The models with $\beta = \pm 1/\sqrt{3}$ on the other hand do not contain particles with exotic electric charges.

3.1 331-models with $\beta = \pm 1/\sqrt{3}$

The models with $\beta = \pm 1/\sqrt{3}$ [152]-[163] have been studied in literature, and they share the same basic structure, altough can differ by some parts of the model. The first true 331-model was presented in 1980 by Singer, Valle and Schechter [152]. This was the first article that noted that the anomaly cancellation does not allow for arbitrary addition of fermion families. There were models previous to this that used $SU(3)_c \times SU(3)_L \times U(1)_X$ gauge group, but it was not used as family structure [135]-[151]. These early models also typically do not take into account the cancellation of chiral anomalies, eventhough the results by Bell, Jackiw and Adler were already known [202], [203].

Here an example "vanilla" $\beta = \pm 1/\sqrt{3}$-model is presented. This model will reveal the usual particle content of these type of 331-models. Variations of this vanilla model exist and they are briefly discussed.

3.1.1 Example model

This model was originally presented in [156]. The electric charge is defined as

$$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X. \tag{3.4}$$

The model with $\beta = +1/\sqrt{3}$ would lead to essentially the same model.

Fermions

The leptons are placed into triplets:

$$\begin{aligned} L_{L,i} &= \begin{pmatrix} \nu_i \\ e_i \\ \nu'_i \end{pmatrix}_L \sim (1, 3, -\frac{1}{3}), \\ e_{R,i} &\sim (1, 1, -1) \quad i = 1, 2, 3. \end{aligned} \tag{3.5}$$

The numbers in the parantheses label the transformation properties under the gauge group $SU(3)_c \times SU(3)_L \times U(1)_X$. The $\nu_{L,i}$ and $e_{L,i}$ are the SM leptons and they transform as an $SU(2)_L$ doublet. The $\nu'_{L,i}$ are new leptons with electric charges 0 and transform as an $SU(2)_L$ singlet.

The cancellation of chiral anomalies requires the number of fermion triplets to be equal to antitriplets. This is achieved by assigning one family into an $SU(3)_L$ triplet and two quark families into $SU(3)_L$ antitriplets. The first quark generation is placed into a triplet and the second and the third into an antitriplet:

$$
\begin{aligned}
&Q_{L,1} = \begin{pmatrix} u_1 \\ d_1 \\ U \end{pmatrix}_L \sim (3,3,\frac{1}{3}), \\
&Q_{L,2} = \begin{pmatrix} d_2 \\ -u_2 \\ D_1 \end{pmatrix}_L, \quad Q_{L,3} = \begin{pmatrix} d_3 \\ -u_3 \\ D_2 \end{pmatrix}_L \sim (3,3^*,0), \\
&u_{R,i} \sim (3,1,\frac{2}{3}), \quad U_R \sim (3,1,\frac{2}{3}), \\
&d_{R,i} \sim (3,1,-\frac{1}{3}), \quad D_{R,1} \sim (3,1,-\frac{1}{3}), \quad D_{R,2} \sim (3,1,-\frac{1}{3}), \quad i = 1,2,3.
\end{aligned} \tag{3.6}
$$

The two upper components of triplets and antitriplets are the SM quarks and transform under $SU(2)_L$. The D_1 and D_2 are new quarks with electric charge $-1/3$ and U with electric charge $2/3$. There are three lepton triplets. When the colour is taken into account, the $Q_{L,1}$ accounts for three triplets. Therefore in total there are six triplets. The antitriplets $Q_{L,2}$ and $Q_{L,3}$ correspond to six antitriplets. Thus there in an equal number of triplets and antitriplets, ensuring the cancellation of pure $SU(3)_L$-anomaly. Also all the other gauge anomalies cancel with this particle content. The gauge anomalies cancel between the generations instead if within a generation, like in the SM.

Scalar sector

The minimal scalar sector of the $\beta = -1/\sqrt{3}$ model consists of three scalar triplets:

$$
\eta = \begin{pmatrix} \eta^+ \\ \eta^0 \\ \eta'^+ \end{pmatrix} \sim (1,3,\frac{2}{3}), \quad \rho = \begin{pmatrix} \rho^0 \\ \rho^- \\ \rho'^0 \end{pmatrix}, \quad \chi = \begin{pmatrix} \chi^0 \\ \chi^- \\ \chi'^0 \end{pmatrix} \sim (1,3,-\frac{1}{3}). \tag{3.7}
$$

This scalar content is minimal in a sense that all the gauge bosons and charged fermions accuire masses at tree-level (all the neutrinos do not acquire masses at tree-level). All the neutral scalars can in principle acquire a non-zero VEV. For simplicity the vacuum is traditionally taken to be:

$$
\langle \eta \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ w \\ 0 \end{pmatrix}, \quad \langle \rho \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix}, \quad \langle \chi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}. \tag{3.8}
$$

The symmetry breaking pattern is as follows:

$$
SU(3)_L \times U(1)_X \stackrel{u}{\to} SU(2)_L \times U(1)_Y \stackrel{v,w}{\to} U(1)_{em}. \tag{3.9}
$$

The VEV u breaks the $SU(3)_L \times U(1)_X \to SU(2)_L \times U(1)_Y$, and v and w break $SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$. It is assumed that v and w are at the electroweak scale and that $u >> v, w$.

All the neutrinos do not acquire masses at tree-level [153]. The reason is that the neutrino mass matrix is antisymmetric:

$$\mathcal{L}_{\text{neutrino mass}} = e_{ij}\epsilon_{\alpha\beta\gamma}\bar{L}^{\alpha}_{L,i}(L^c_{L,j})^{\beta}\langle\eta^*\rangle^{\gamma} \supset \frac{w}{\sqrt{2}}e_{ij}\overline{(\nu_L)^c}\nu'_L + h.c. \tag{3.10}$$

The neutrino mass matrix $(w/\sqrt{2})e_{ij}$ is antisymmetric due to the presence of totally antisymmetric tensor in Eq. (3.10). This is a problem as it will have one zero eigenvalue and the other two will be degenerate. Radiative corrections are needed to break the degeneracy and to lift the one mass from zero. Both neutrino particle and antiparticle are placed in a same multiplet, and radiative corrections can induce a lepton number violating Majorana mass term. This was first observed by Wolfenstein [217]-[220]. The present model does not contain right-handed neutrino singlets. If the right-handed neutrino singlets are added, all the neutrinos will acquire tree-level masses [153]. The neutrino masses have also been studied recently in $\beta = \pm 1/\sqrt{3}$ -setting [185]-[188].

Gauge sector

The gauge sector of a 331-model contains five additional gauge bosons compared to the SM. The covariant derivative for a triplet is:

$$D_\mu = \partial_\mu - ig_3\sum_{a=1}^{8} T_a W_{a\mu} - ig_x X B_\mu, \tag{3.11}$$

where g_3 and g_x are the $SU(3)_L$ and $U(1)_X$ gauge couplings, respectively. The $SU(3)_L$ generators are given as: $T_a = \lambda_a/2$, where λ_a are the Gell-Mann matrices. The $SU(3)_L$ gauge bosons are:

$$\sum_{a=1}^{8} T_a W_{a\mu} = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{1}{\sqrt{2}}W_{3\mu} + \frac{1}{\sqrt{6}}W_{8\mu} & W^+_\mu & X^0_\mu \\ W^-_\mu & -\frac{1}{\sqrt{2}}W_{3\mu} + \frac{1}{\sqrt{6}}W_{8\mu} & V^-_\mu \\ X^{0*}_\mu & V^+_\mu & -\frac{2}{\sqrt{6}}W_{8\mu} \end{pmatrix}, \tag{3.12}$$

where the following notation is used:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_{1\mu} \mp iW_{2\mu}),$$
$$V_\mu^\mp = \frac{1}{\sqrt{2}}(W_{6\mu} \mp iW_{7\mu}),$$
$$X_\mu^0 = \frac{1}{\sqrt{2}}(W_{4\mu} - iW_{5\mu}).$$

The fields in the diagonal $W_{3\mu}$, $W_{8\mu}$ and B_μ will form neutral mass eigenstates: photon, Z-boson and new heavy gauge boson Z'. The off-diagonal gauge boson $W_\mu^\pm$ is identified as the $W_\mu^\pm$-boson of the SM. The off-diagonal $V_\mu^\pm$ is a new singly charged gauge boson. This gauge boson will not mix with the $W_\mu^\pm$ with the vacuum presented in Eq. (3.8). Most general vacuum would have non-zero VEV, $\langle \rho'^0 \rangle \neq 0$, and in this case the $W_\mu^\pm$ and $V_\mu^\pm$ will mix. The X_μ^0 field is a *neutral non-Hermitian gauge boson.* This will not mix with the other neutral gauge bosons with vacuum in Eq. (3.8). The masses of the new gauge bosons are proportional to the $SU(3)_L \times U(1)_X$-breaking VEV u and the new gauge bosons are therefore very heavy, at least multiple TeV.

3.1.2 Variants of the $\beta = \pm 1/\sqrt{3}$

There exist variants of the basic $\beta = \pm 1/\sqrt{3}$ model presented in the previous section. These include less than minimal models. The basic model contain three scalar triplets, which generate all the gauge boson masses and charged fermion masses in the spontaneous symmetry breaking. The breaking of gauge $SU(3)_L \times U(1)_X$ symmetry to $U(1)_{em}$, however, requires only two scalar triplets. A $\beta = \pm 1/\sqrt{3}$ model with only two scalar triplets is studied in [221]-[227], [201]. The models with this reduced scalar sector do not generate all the charged fermion masses at tree-level and rely on radiative corrections to generate the masses of the lightest charged fermions. This fact is often argued to partially explain the fermion mass hierarchy. In the $\beta = \pm 1/\sqrt{3}$ models with two scalar triplets [221], [222], some of the neutrino masses are generated at tree-level, while the masses of the lightest quarks are generated radiatively, even though the lightest quark is many orders of magnitude heavier than the heaviest neutrino. Therefore the fermion mass hierarchy problem is solved only partially.

Another variant is to change the lepton representations. The $\beta = -1/\sqrt{3}$ ($\beta = +1/\sqrt{3}$) models traditionally place the leptons into triplet (antitriplet) [152]-[163]. This way the new lepton is a neutral neutrino-like field. If in the $\beta = -1/\sqrt{3}$ ($\beta = +1/\sqrt{3}$) models the leptons are instead placed in the antitriplet (triplet), the new lepton field is charged [158],[162].

One can also choose which quark generation is treated differently from the other two generations. In the example model it was the first generation. If the third generation is treated diffrently, it will acquire masses through a different VEV than the other two. This VEV could be tuned to be larger than the other $SU(2)_L \times U(1)_Y$-breaking VEV. This could be used to explain the heaviness of the top quark.

3.2 331-models with $\beta = \pm\sqrt{3}$

The models based on $\beta = \pm\sqrt{3}$ [164]-[175] have been extensively studied in the literature. The 331-models based on $\beta = \pm\sqrt{3}$ were first introduced in [164], [165]. The $\beta = \pm\sqrt{3}$-models are perhaps the simplest gauge extension of the SM containing doubly charged gauge bosons [164]. Here the particle content of the model based on $\beta = \pm\sqrt{3}$ is presented. Variants of this model exist, but this illustrates the common properties of the models of this type.

3.2.1 Particle content

This model was originally presented in [166]. The electric charge is defined as,

$$Q = T_3 - \sqrt{3}T_8 + X. \tag{3.13}$$

The model with $\beta = +\sqrt{3}$ would lead to essentially the same model.

Fermions

The leptons are placed into triplets:

$$L_{L,i} = \begin{pmatrix} \nu_{L,i} \\ e_{L,i} \\ (e_{R,i})^c \end{pmatrix} \sim (1, 3, 0). \tag{3.14}$$

All the SM leptons $\nu_{L,i}$, $e_{L,i}$, $e_{R,i}$ are in the same multiplet and there are no new lepton fields as in the $\beta = \pm 1/\sqrt{3}$-case. At first this seems really economical, but this has severe consequences as we shall see. The present model does not contain right-handed neutrino singlets, and therefore the neutrinos will remain massless. The incorporation of the right-handed neutrino singlets will generate neutrino masses.

The SM quarks are assigned into the triplets and antitriplets similarly to the $\beta = -1/\sqrt{3}$-model presented previously. The quark multiplets now differ by their third component compared to the $\beta = -1/\sqrt{3}$-case.

$$Q_{L,1} = \begin{pmatrix} u_1 \\ d_1 \\ J_1 \end{pmatrix}_L \sim (3,3,\frac{2}{3}), \tag{3.15}$$

$$Q_{L,2} = \begin{pmatrix} d_2 \\ -u_2 \\ J_2 \end{pmatrix}_L, \quad Q_{L,3} = \begin{pmatrix} d_3 \\ -u_3 \\ J_3 \end{pmatrix}_L \sim (3,3^*,-\frac{1}{3}), \tag{3.16}$$

$$u_{R,i} \sim (3,1,\frac{2}{3}), \quad d_{R,i} \sim (3,1,-\frac{1}{3}), \tag{3.17}$$
$$J_{R,1} \sim (3,1,\frac{5}{3}), \quad J_{R,2} \sim (3,1,-\frac{4}{3}), \quad J_{R,3} \sim (3,1,-\frac{4}{3}), \quad i = 1,2,3.$$

Again the two upper components of triplets and antitriplets are the SM quarks. The J_1, J_2 and J_3 are new quarks with exotic electric charges. The J_1 has an electric charge $+5/3$, and J_2 and J_3 have electric charge $-4/3$. There are equal number of triplets and antitriplets, which ensures the cancellation of pure $SU(3)_L$ anomaly. Also all the other anomalies cancel. The exotic quarks have different electric charges than the SM quarks and there is no mixing between exotic and SM quarks, unlike in the $\beta = \pm 1/\sqrt{3}$-case.

Scalar sector

The scalar sector of $\beta = -\sqrt{3}$ model requires at least 3 scalar triplets:

$$\eta = \begin{pmatrix} \eta^0 \\ \eta_1^- \\ \eta_2^+ \end{pmatrix} \sim (1,3,0), \quad \rho = \begin{pmatrix} \rho^+ \\ \rho^0 \\ \rho^{++} \end{pmatrix} \sim (1,3,1), \tag{3.18}$$
$$\chi = \begin{pmatrix} \chi^- \\ \chi^{--} \\ \chi^0 \end{pmatrix} \sim (1,3,-1).$$

The scalar sector contains doubly charged scalars unlike the SM. The most general electric charge conserving vacuum is:

$$\langle\eta\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v \\ 0 \\ 0 \end{pmatrix}, \quad \langle\rho\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ w \\ 0 \end{pmatrix}, \quad \langle\chi\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}. \tag{3.19}$$

Also $\beta = \pm\sqrt{3}$ models have been studied where the scalar sector consists of only two scalar triplets [228], [229], [198]. These less than minimal models rely on effective operators to generate masses for many of the particles. The scalar content in Eq. (3.18) is enough to break the gauge symmetries and to generate the gauge boson masses. Also all the quarks acquire tree-level masses. There is a problem with the charged leptons: electron is massless at tree-level. Furthermore the muon and tau masses are degenerate at tree-level. This is phenomenologically unacceptable. This is due to antisymmetric charged lepton mass matrix. Situation is similar to the mass of neutrinos in the $\beta = \pm 1/\sqrt{3}$-case. The problem with the charged lepton masses is solved with a high prize: scalar sector needs to be extended with a scalar sextet to generate general mass matrix for charged leptons instead of antisymmetric one. Historically the first article studying $\beta = \pm\sqrt{3}$ by Pisano and Pleitez was missing the sextet and the charged lepton masses were unacceptable [164]. This was soon remedied by Frampton [165]. Pisano, Pleitez and Tonasse later also studied the radiative generation of lepton masses in a model without the scalar sextet [173]. The scalar antisextet is introduced:

$$S = \begin{pmatrix} \sigma_1^0 & h_2^+ & h_1^- \\ h_2^+ & H_1^{++} & \sigma_2^0 \\ h_1^- & \sigma_2^0 & H_2^{--} \end{pmatrix} \sim (1, 6^*, 0). \tag{3.20}$$

The vacuum of S is chosen to be:

$$\langle S \rangle = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & v' \\ 0 & v' & 0 \end{pmatrix}. \tag{3.21}$$

The VEV u will break the $SU(3)_L \times U(1)_X \to SU(2)_L \times U(1)_Y$ and it is assumed that $u >> v, w, v'$. The VEVs v, w and v' break the $SU(2)_L \times U(1)_Y \to U(1)_{\text{em}}$ and they are taken to be of the order of electroweak scale.

Gauge sector

Let us study the gauge bosons of the model by examining the covariant derivative presented in Eq. (3.11). The $SU(3)_L$ gauge bosons are:

$$\sum_{a=1}^{8} T_a W_{a\mu} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} W_{3\mu} + \frac{1}{\sqrt{6}} W_{8\mu} & W_\mu^+ & V_\mu^- \\ W'^-_\mu & -\frac{1}{\sqrt{2}} W_{3\mu} + \frac{1}{\sqrt{6}} W_{8\mu} & U_\mu^{--} \\ V_\mu^+ & U_\mu^{++} & -\frac{2}{\sqrt{6}} W_{8\mu} \end{pmatrix},$$

where we have denoted;

$$
\begin{aligned}
W_\mu^{\pm} &= \frac{1}{\sqrt{2}}(W_{1\mu} \mp iW_{2\mu}), \\
V_\mu^{\mp} &= \frac{1}{\sqrt{2}}(W_{4\mu} \mp iW_{5\mu}), \\
U_\mu^{\mp\mp} &= \frac{1}{\sqrt{2}}(W_{6\mu} \mp iW_{7\mu}).
\end{aligned}
$$

The diagonal gauge bosons $W_{3\mu}$, $W_{8\mu}$ and B_μ produce the neutral physical gauge bosons, photon, Z_μ^0-boson and the new heavy gauge boson Z'^0_μ. The off-diagonal $W_\mu^{\pm}$ are identified as the SM $W_\mu^{\pm}$ gauge bosons. The $V_\mu^{\mp}$ are a new singly charged gauge bosons. Finally the model contains doubly charged gauge bosons $U_\mu^{\mp\mp}$. The doubly charged gauge bosons are *bileptons*, that will decay into two same sign leptons. Bileptons are also present in $SU(15)$ Grand Unified Theories [230]-[233], but 331-models offer a much simpler framework for them. The collider phenomenology of the bileptons is studied in [234].

3.3 Discussion

The 331-models predict the existense of 3 families, but only if certain conditions are imposed. One of these conditions was that of minimality: one wants to introduce as few new particles as possible. In the fermion sector this is accomplished by having one left-handed multiplet for one generation. This way there is at most one new fermion per triplet/antitriplet. With this constraint the anomalies cancel between the generations instead within a generation like in the SM. The situation changes if the minimality condition is lifted. The anomalies can be made to cancel within a generation in a 331-model when one is free to assign fermions to the representations however desired. These are called sequential 331-models [235], [236]. These sequantial models cancel anomalies within a generation, and do not predict number of fermion families. They are also substantially more complicated as the number of new fermion fields is increased. The traditional 331-models treat the lepton generations equally but quark generations differently. In some models also the lepton generations are treated differently [162],[237],[238],[239]. These models also do not predict the number of families.

One motivation for the sequential 331-models comes when one attempts to embed 331-models into some Grand Unified Theory-models [230]-[233], [240]-[244]. The cancellation of anomalies forces the quark families to appear in different representations in traditional 331-models. Due to this the traditional 331-models are difficult to embed into the traditional Grand Unified Theories. The 331-model can be easily embeded to GUT when the model is sequential. The 331-models have been embedded to E_6 [235], $SU(6)$ [162] and $SU(6) \times U(1)_X$ [245]. Also string completions of the 331-models have been suggested [246].

The 331-models explain neatly the number of fermion families. They however leave some problems behind, some of which are their doing and some that are not. The cancellation of gauge anomalies requires one quark generation to be placed into a different representation than the rest. This leads to scalar mediated FCNCs of quarks at tree-level. This serious problem is omitted in the literature by appealing to special structure of Yukawa matrices to magically cancel the dangerous FCNC contributions [152], [153], [154], [158], [159], [164], [171]. The fermion mass hierarchy is also left unexplained in the 331-models, and this is not a fault of 331-models, as this problem is already present in the SM. In fact the 331-models make the case for partial explanation of the fermion mass hierarchy. If the third quark generation is the one treated differently from the rest, then the top quark will receive its mass from a different VEV than the other quarks. By tuning that VEV larger than the other VEVs, the exceptional heaviness of the top quark can be explained [165]. This therefore partially explains the hierarchy of fermion masses.

The flavour problem has two parts: the fermion family number problem and the fermion mass hierarchy problem. The 331-models solve the former and potentially partially solve the latter. This sounds quite interesting: 331-models almost offer the solution to the flavour problem. The inevitable presence of scalar mediated tree-level FCNCs of quarks, however, dims this.

Chapter 4

331-models with Froggatt-Nielsen mechanism

The 331-models explain the number of fermion families, but do not explain the fermion mass hierarchy in a satisfactory manner. This is a pity, as half of the flavour problem is already solved. One would be tempted to extend the 331-models with the Froggatt-Nielsen mechanism. This would give the solution to the fermion mass hierarchy, and the flavour problem would be solved. The scalar sector of the 331-models is, however, notoriously complicated. Even the simpler $\beta = \pm 1/\sqrt{3}$-models have minimally three scalar triplets. The situation in $\beta = \pm\sqrt{3}$-models is even worse, as an additional scalar sextet is required.[1] The introduction of the traditional FN-mechanism would extend the 331-model with an additional complex scalar, which would make the already complicated scalar sector even more so.

The extension of the scalar sector is, however, not neccesary. The Froggatt-Nielsen mechanism can be economically incorporated into the 331-models, by using the already existing scalar sector of the 331-models (FN331-models) [213], [214]. The minimal scalar sector of the $\beta = \pm 1/\sqrt{3}$-models contains two scalar triplets in the same representation, as can be seen from the Eq. (3.7). The FN331-models are based on $\beta = \pm 1/\sqrt{3}$-models, due to this special propery of their scalar sector. The two scalar triplets in the same representation in the Eq. (3.7) can be combined into a gauge singlet:

$$\rho^\dagger \chi \sim (1, 1, 0). \tag{4.1}$$

This combination can carry a non-zero global $U(1)$-charge, since ρ and χ are different fields. This allows it to play the role of the flavon in the FN mechanism. This is intriguing: the 331-models known for their ability to explain the number of families can so

[1]The scalar sector of 331-models have been studied in [163], [175], [247], [248].

easily incorporate the FN-mechanism, the best method for generating the fermion mass hierarchy. The minimal scalar sector of the $\beta = \pm\sqrt{3}$-models do not contain two scalar triplets in the same representation, as seen in the Eq. (3.18), and the FN mechanism cannot be economically incorporated into them. The idea of using the existing particle content of the model as the effective flavon has been done in 2HDM [249], [250], where the flavon is constructed from two Higgs doublets.

4.1 FN331-model

The FN331-model is based on the models with $\beta = \pm 1/\sqrt{3}$. The value is now chosen to be $\beta = -1/\sqrt{3}$ for concreteness. The particle content of the model is given in the Eqs. (3.5), (3.6), (3.7), (3.11) and (3.12). The fermions and scalars are charged under global $U(1)_{\text{FN}}$ symmetry to forbid the usual Yukawa couplings. The scalar triplets are given the FN-charges in Table 4.1.

Particle	η	ρ	χ
FN-charge	-1	1	0

Table 4.1: The FN $U(1)$ charges of the scalar triplets

The vacuum of the scalar triplets has to be altered from Eq. (3.8) in order FN mechanism to work. In the FN331-model the most general electric charge conserving vacuum is chosen:

$$\langle\eta\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v' \\ 0 \end{pmatrix}, \quad \langle\rho\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} v_1 \\ 0 \\ v_2 \end{pmatrix}, \quad \langle\chi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ 0 \\ u \end{pmatrix}. \tag{4.2}$$

The VEVs v_2 and u break the $SU(3)_L \times U(1)_X$ to the SM and VEVs v' and v_1 break the electroweak symmetry to the electromagnetism:

$$SU(3)_L \times U(1)_X \stackrel{u,v_2}{\to} SU(2)_L \times U(1)_Y \stackrel{v',v_1}{\to} U(1)_{em}. \tag{4.3}$$

The hierarchy, $v_2, u >> v', v_1$, is assumed. The upper component in χ is neutral and could in principle be non-zero. One can, however, perform a $SU(3)_L$-rotation to the vacuum to make the VEV of the first component of $\langle\chi\rangle$ zero, without the loss of generality [251]. The effective flavon now has a non-zero VEV:

$$\langle\rho^\dagger\chi\rangle = \frac{uv_2}{2}. \tag{4.4}$$

The combination $\rho^\dagger\chi$ has a FN-charge -1.

4.1.1 The Froggatt-Nielsen mechanism in the 331-framework

The FN mechanism with the effective flavon works similarly to the elementary flavon case. The effective operator generating the 331-Yukawa couplings is analogous to the Eq. (2.2):

$$\mathcal{L} \supset (c_s^f)_{ij} \left(\frac{\rho^\dagger \chi}{\Lambda^2}\right)^{(n_f^s)_{ij}} \bar{\psi}^f_{L,i} S f_{R,j} + h.c., \tag{4.5}$$

where the S denotes any of the three scalar triplets η, ρ or χ and $(c_s^f)_{ij}$ is a dimensionless order-one number. The $\psi^f_{L,i}$ and $f_{R,j}$ represent the fermion triplets, anti-triplets and singlets that are presented in Eqs. (3.5) and (3.6). The power $(n_f^s)_{ij}$ is determined by the FN charge assignment given in the Table 4.2:

$$(n_f^s)_{ij} = \left[q(\bar{\psi}^f_{L,i}) + q(f_{R,j}) + q(S)\right]. \tag{4.6}$$

Particle	$(\psi^f_{L,i})^c$	$f_{R,i}$	S
FN charge	$q(\bar{\psi}^f_{L,i})$	$q(f_{R,i})$	q_S

Table 4.2: The FN charges of fermions and the scalar fields.

The $SU(3)_L \times U(1)_X$-symmetry breaks as the scalar triplets ρ and χ acquire VEVs. The $U(1)_{\mathrm{FN}}$ breaks at the same time and the usual 331 Yukawa-terms are generated as effective couplings:

$$\begin{aligned}\mathcal{L} &\supset (c_s^f)_{ij} \left(\frac{(\rho + \langle\rho\rangle)^\dagger (\chi + \langle\chi\rangle)}{\Lambda^2}\right)^{(n_f^s)_{ij}} \bar{\psi}^f_{L,i}(S + \langle S\rangle) f_{R,j} + h.c. \qquad (4.7)\\ &= (y_s^f)_{ij} \bar{\psi}^f_{L,i}(S + \langle S\rangle) f_{R,j} + (n_f^s)_{ij}(y_s^f)_{ij}\left[\frac{{\rho'}^{0*}}{v_2} + \frac{{\chi'}^0}{u} + \frac{v_1 \chi^0}{v_2 u}\right]\sqrt{2}\bar{\psi}^f_{L,i}\langle S\rangle f_{R,j} + h.c. + \cdots,\end{aligned}$$

where only the renormalizable contributions are kept. The Yukawa couplings are defined as:

$$(y_s^f)_{ij} = (c_s^f)_{ij}\left(\frac{v_2 u}{2\Lambda^2}\right)^{(n_f^s)_{ij}} \equiv (c_s^f)_{ij}\epsilon^{(n_f^s)_{ij}}. \tag{4.8}$$

The first term in Eq. (4.7) gives the usual Yukawa terms of the model. The second term is a flavour violating part characteristic to Froggatt-Nielsen mechanism. We have set the expansion parameter ϵ to the Cabibbo angle $\epsilon = 0.23$.

4.1.2 Quark mass matrices

The traditional 331-models have scalar mediated quark FCNCs at tree-level. The FN331-model is no exception. The difference is that FN331-model offers a concrete suppression mechanism that is elaborated shortly. The charged leptons couple to only one scalar triplet. Their mass matrix is therefore proportional to the Yukawa matrix and there will be no flavour violating effects from neutral scalars for the charged leptons [213], [214]. The up- and down-type quarks couple to multiple scalar triplets due to unequal treatment of quark generations. The up- and down-type quark Yukawa couplings are:

$$\begin{aligned}\mathcal{L}_{up} &= \sum_{\gamma=1}^{4}(y^u_\rho)_{1\gamma}\bar{Q}'_{L,1}\rho\ u'_{R,\gamma} + \sum_{\gamma=1}^{4}(y^u_\chi)_{1\gamma}\bar{Q}'_{L,1}\chi\ u'_{R,\gamma} \\ &+ \sum_{\alpha=2}^{3}\sum_{\gamma=1}^{4}(y^u_{\eta^*})_{\alpha\gamma}\bar{Q}'_{L,\alpha}\eta^*\ u'_{R,\gamma} + h.c.,\end{aligned} \tag{4.9}$$

and,

$$\begin{aligned}\mathcal{L}_{down} &= \sum_{\gamma=1}^{5}(y^d_\eta)_{1\gamma}\bar{Q}'_{L,1}\eta\ d'_{R,\gamma} + \sum_{\alpha=2}^{3}\sum_{\gamma=1}^{5}(y^d_{\rho^*})_{\alpha\gamma}\bar{Q}'_{L,\alpha}\rho^*\ d'_{R,\gamma} \\ &+ \sum_{\alpha=2}^{3}\sum_{\gamma=1}^{5}(y^d_{\chi^*})_{\alpha\gamma}\bar{Q}'_{L,\alpha}\chi^*\ d'_{R,\gamma} + h.c.,\end{aligned} \tag{4.10}$$

where $u'_R = (u'_{R,1}, u'_{R,2}, u'_{R,3}, U'_R)$ and $d'_R = (d'_{R,1}, d'_{R,2}, d'_{R,3}, D'_{R,1}, D'_{R,2})$. The Yukawa couplings are qiven by the FN-mechanism:

$$\left\{\begin{array}{l}(y^u_\rho)_{1\gamma} = (c^u_\rho)_{1\gamma}\epsilon^{q(\bar{Q}_{L,1})+q(u_{R,\gamma})+q(\rho)} \\ (y^u_{\eta^*})_{\alpha\gamma} = (c^u_{\eta^*})_{\alpha\gamma}\epsilon^{q(\bar{Q}_{L,\alpha})+q(u_{R,\gamma})+q(\eta^*)} \\ (y^u_\chi)_{1\gamma} = (c^u_\chi)_{1\gamma}\epsilon^{q(\bar{Q}_{L,1})+q(u_{R,\gamma})+q(\chi)}\end{array}\right. \quad \left\{\begin{array}{l}(y^d_\eta)_{1\gamma} = (c^d_\eta)_{1\gamma}\epsilon^{q(\bar{Q}_{L,1})+q(d_{R,\gamma})+q(\eta)} \\ (y^d_{\rho^*})_{\alpha\gamma} = (c^d_{\rho^*})_{\alpha\gamma}\epsilon^{q(\bar{Q}_{L,\alpha})+q(d_{R,\gamma})+q(\rho^*)} \\ (y^d_{\chi^*})_{\alpha\gamma} = (c^d_{\chi^*})_{\alpha\gamma}\epsilon^{q(\bar{Q}_{L,\alpha})+q(d_{R,\gamma})+q(\chi^*)}\end{array}\right. \tag{4.11}$$

where $\alpha = 2, 3$ and $\gamma = 1, 2, 3, 4$. One might be tempted to introduce a disctere symmetry in order to couple only one scalar triplet to a given quark type as is typically done in 2HDMs to avoid tree-level FCNCs [252], [253]. This is not an attractive option as this interferes with the mass generation of quarks: all the Yukawa couplings in Eqs. (4.9) and (4.10) are required in order all the quarks to acquire tree-level masses.

The up- and down-type quark masses are generated by the terms in the Eqs. (4.9) and (4.10) as the scalars acquire VEVs.

$$\mathcal{L}_{quark-mass} = \bar{u}'_L m_u u'_R + \bar{d}'_L m_d d'_R + h.c., \tag{4.12}$$

The exotic quarks will mix with the SM quarks. The mass scale of the exotic quarks is set by the $SU(3)_L \times U(1)_X$-breaking scale. This introduces an additional source of hierarchy to the quark mass matrices. In the traditional FN-mechanims the only source of hierarchy is coming from the FN-charge assignment. The quark mass matrices can be written in a way where the effect of the both sources of hierarchy is transparent. The up-type quark mass matrix is:

$$
\begin{aligned}
(m_u)_{1\gamma} &= \frac{v'}{\sqrt{2}} \left[\frac{v_1}{v'}(c^u_\rho)_{1\gamma}\right] \epsilon^{a^u_1 + q(u_{R,\gamma})}, \\
(m_u)_{2\gamma} &= \frac{v'}{\sqrt{2}} \left[-(c^u_{\eta^*})_{2\gamma}\right] \epsilon^{a^u_2 + q(u_{R,\gamma})}, \\
(m_u)_{3\gamma} &= \frac{v'}{\sqrt{2}} \left[-(c^u_{\eta^*})_{3\gamma}\right] \epsilon^{a^u_3 + q(u_{R,\gamma})}, \\
(m_u)_{4\gamma} &= \frac{v'}{\sqrt{2}} \left[(c^u_\rho)_{1\gamma}\epsilon^{q(\rho)-q(\chi)} + (c^u_\chi)_{1\gamma}\epsilon^{(\log\epsilon)^{-1}\log(u/v_2)}\right] \epsilon^{a^u_4 + q(u_{R,\gamma})}.
\end{aligned}
$$

The quantities in square brackets are order-one numbers. The hierarchy of the mass matrix is therefore completely given in the powers of ϵ. The difference between $SU(3)_L \times U(1)_X$ and $SU(2)_L \times U(1)_Y$-breaking scales manifests itself as effective left-handed charges a^u_γ:

$$
\begin{aligned}
a^u_1 &= q(\bar{Q}_{L,1}) + q(\rho), \\
a^u_2 &= q(\bar{Q}_{L,2}) + q(\eta^*), \\
a^u_3 &= q(\bar{Q}_{L,3}) + q(\eta^*), \\
a^u_4 &= (\log\epsilon)^{-1}\log\left(\frac{v_2}{v'}\right) + q(\bar{Q}_{L,1}) + q(\chi).
\end{aligned} \tag{4.13}
$$

Similarly the down-type quark mass matrix is given by:

$$
\begin{aligned}
(m_d)_{1\gamma} &= \frac{v'}{\sqrt{2}} \left[(c^d_\eta)_{1\gamma}\right] \epsilon^{a^d_1 + q(d_{R,\gamma})}, \\
(m_d)_{2\gamma} &= \frac{v'}{\sqrt{2}} \left[\frac{v_1}{v'}(c^d_{\rho^*})_{2\gamma}\right] \epsilon^{a^d_2 + q(d_{R,\gamma})}, \\
(m_d)_{3\gamma} &= \frac{v'}{\sqrt{2}} \left[\frac{v_1}{v'}(c^d_{\rho^*})_{3\gamma}\right] \epsilon^{a^d_3 + q(d_{R,\gamma})}, \\
(m_d)_{4\gamma} &= \frac{v'}{\sqrt{2}} \left[(c^d_{\rho^*})_{2\gamma} + (c^d_{\chi^*})_{2\gamma}\epsilon^{(\log\epsilon)^{-1}\log(u/v_2)+q(\chi^*)-q(\rho^*)}\right] \epsilon^{a^d_4 + q(d_{R,\gamma})}, \\
(m_d)_{5\gamma} &= \frac{v'}{\sqrt{2}} \left[(c^d_{\rho^*})_{3\gamma} + (c^d_{\chi^*})_{3\gamma}\epsilon^{(\log\epsilon)^{-1}\log(u/v_2)+q(\chi^*)-q(\rho^*)}\right] \epsilon^{a^d_5 + q(d_{R,\gamma})},
\end{aligned}
$$

where the effective left-handed charges are:

$$\begin{aligned}
a_1^d &= q(\bar{Q}_{L,1}) + q(\eta), \\
a_2^d &= q(\bar{Q}_{L,2}) + q(\rho^*), \\
a_3^d &= q(\bar{Q}_{L,3}) + q(\rho^*), \\
a_4^d &= (\log\epsilon)^{-1}\log\left(\frac{v_2}{v'}\right) + q(\bar{Q}_{L,2}) + q(\rho^*), \\
a_5^d &= (\log\epsilon)^{-1}\log\left(\frac{v_2}{v'}\right) + q(\bar{Q}_{L,3}) + q(\rho^*).
\end{aligned} \tag{4.14}$$

The quark FCNCs acquire suppression when the following hierarchy in the quark mass matrices is assumed:

$$m_{i,j}^q \leq m_{i+1,j}^q, \tag{4.15}$$

where $q = u, d$. This can be accomplished by choosing the left-handed quark FN-charges as:

$$(Q_{L,3}^c) \leq (Q_{L,2}^c) \leq (Q_{L,1}^c), \tag{4.16}$$

and by demanding that the $SU(3)_L \times U(1)_X$-breaking scale is sufficiently high. The condition in Eq. (4.15) ensures that the left-handed quark diagonalization matrices are close to unity:

$$(U_L^q)_{ij} \sim \epsilon^{|a_i^q - a_j^q|}. \tag{4.17}$$

The left-handed quark FN charges are chosen to be $q(Q_{L,1}^c) = 2$, $q(Q_{L,2}^c) = 1$, $q(Q_{L,3}^c) = -1$ for concreteness. This choice produces the correct CKM-matrix texture given in Eq. (2.14).

4.1.3 Higgs mediated quark FCNCs

There are many potentially dangerous quark flavour changing processes that acquire contributions from the scalars of the 331-model including: neutral meson[2] mixing, leptonic decay of neutral meson, $M^0 \to l_i^+ l_i^-$, radiative B-meson decay, $\bar{B}_d^0 \to X_s^0\gamma$, and top quark decays, $t \to hc$ and $t \to q\gamma$. The neutral meson mixing is often the most constraining process. In SM the neutral meson mixing is a 1-loop process. In the 331-models it takes place at tree-level, making it potentially very dangerous. The lightest neutral scalar is presumably the 125 GeV Higgs, so there is not much suppression coming from the mediator mass. The flavour violating couplings themselves have to be sufficiently small. The magnitude of these couplings can be directly estimated in the FN-mechanism.

[2]The neutral mesons and their quark content: $K^0 = d\bar{s}$, $B_d^0 = d\bar{b}$, $B_s^0 = s\bar{b}$, $D^0 = c\bar{u}$.

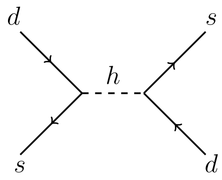

Figure 4.1: The 125 GeV Higgs mediates neutral kaon mixing at tree-level. The other neutral meson mixing diagrams are obtained renaming the external legs.

The FN331-models contain six physical neutral scalars: Four CP-even scalars h, H_1, H_2 and H_3, and two CP-odd scalars A_1 and A_2. The h is identified as the 125 GeV Higgs. The masses of the scalars H_1, H_2, H_3 and A_1 are proportional to $SU(3)_L \times U(1)_X$-breaking VEVs u and v_2, and therefore they are very heavy. The A_2 is the pseudo-Goldstone boson associated with the breaking of the global $U(1)_{\rm FN}$-symmetry. The mass of the pseudo-Goldstone A_2 is controlled by the soft-breaking term. The mass scale of A_2 is independent of the $SU(3)_L \times U(1)_X$-breaking scale. In [213], [214] the mass of pseudo-Goldstone A_2 is taken to be several TeVs. This way h is the only dangerous neutral scalar mediating quark FCNCs.

The mass eigenstate Higgs h couples to the mass eigenstate quark as:

$$\mathcal{L}_{\text{quark-Higgs}} = \frac{1}{\sqrt{2}}\bar{u}_L(\Gamma^u_h)u_R\, h + \frac{1}{\sqrt{2}}\bar{d}_L(\Gamma^d_h)d_R\, h + h.c., \tag{4.18}$$

where the physical Yukawa couplings can be written as:

$$\begin{aligned}(\Gamma^u_h)_{ij} &= \sqrt{2}\frac{m_j}{v_{SM}}\Big[\delta_{ij} + \alpha_1 (U^u_L)_{i1}(U^{u\dagger}_L)_{1j} - (U^u_L)_{i4}(U^{u\dagger}_L)_{4j} \\ &\quad + \alpha_2 (U^u_L)_{i1}(U^{u\dagger}_L)_{4j} + \alpha_3 (U^u_L)_{i4}(U^{u\dagger}_L)_{1j}\Big],\end{aligned} \tag{4.19}$$

and,

$$\begin{aligned}(\Gamma^d_h)_{ij} = \sqrt{2}\frac{m_j}{v_{SM}}\Big\{&\delta_{ij} + \beta_1\Big[(U^u_L)_{i2}(U^{u\dagger}_L)_{2j} + (U^u_L)_{i3}(U^{u\dagger}_L)_{3j}\Big] \\ &- \Big[(U^d_L)_{i4}(U^{d\dagger}_L)_{4j} + (U^d_L)_{i5}(U^{d\dagger}_L)_{5j}\Big] \\ &+\beta_2\Big[(U^d_L)_{i2}(U^{d\dagger}_L)_{4j} + (U^d_L)_{i3}(U^{d\dagger}_L)_{5j}\Big] \\ &+\beta_3\Big[(U^d_L)_{i4}(U^{d\dagger}_L)_{2j} + (U^d_L)_{i5}(U^{d\dagger}_L)_{3j}\Big]\Big\},\end{aligned} \tag{4.20}$$

where α_i and β_i are $\mathcal{O}(v_{light}/v_{heavy})$. The v_{light} is the $SU(2)_L \times U(1)_Y$-breaking scale and v_{heavy} is the $SU(3)_L \times U(1)_X$-breaking scale. Most of the off-diagonal terms in Eqs. (4.19) and (4.20) are suppressed by the ratio of $SU(2)_L \times U(1)_Y$ and $SU(3)_L \times U(1)_X$-breaking scales. These terms can be small by increasing the scale of $SU(3)_L \times U(1)_X$-breaking. Only the third term is without this suppression. The $i \neq j$-part of the third term in Eqs. (4.19) and (4.20) is small due to the almost diagonal nature of the left-handed rotation matrices U_L^q.

The textures of the physical Yukawa couplings can be estimated as:

$$\Gamma_h^u \sim \begin{pmatrix} y_u & y_c[\epsilon^1\delta] & y_t[\epsilon^{-2}\delta^2] & \frac{m_U}{v_{heavy}} \\ y_u[\epsilon^1\delta] & y_c & y_t[\epsilon^{-2}\delta^2] & \frac{m_U}{v_{heavy}} \\ y_u[\epsilon^{-2}\delta^2] & y_c[\epsilon^{-2}\delta^2] & y_t & \frac{m_U}{v_{heavy}}\epsilon^{-2} \\ y_u[\delta] & y_c[\delta] & y_t[\epsilon^{-2}\delta] & \frac{m_U}{v_{heavy}}\epsilon^{-4}\delta \end{pmatrix}, \tag{4.21}$$

and,

$$\Gamma_h^d \sim \begin{pmatrix} y_d & y_s[\epsilon^1\delta] & y_b[\epsilon^{-1}\delta^2] & \frac{m_{D_1}}{v_{heavy}}\epsilon^1 & \frac{m_{D_2}}{v_{heavy}}\epsilon^3 \\ y_d[\epsilon^1\delta] & y_s & y_b[\epsilon^{-2}\delta^2] & \frac{m_{D_1}}{v_{heavy}} & \frac{m_{D_2}}{v_{heavy}}\epsilon^2 \\ y_d[\epsilon^{-1}\delta^2] & y_s[\epsilon^{-2}\delta^2] & y_b & \frac{m_{D_1}}{v_{heavy}}\epsilon^{-2} & \frac{m_{D_2}}{v_{heavy}} \\ y_d[\epsilon^1\delta] & y_s[\delta] & y_b[\epsilon^{-2}\delta] & \frac{m_{D_1}}{v_{heavy}}\delta\epsilon^{-4} & \frac{m_{D_2}}{v_{heavy}}\epsilon^{-2}\delta \\ y_d[\epsilon^3\delta] & y_s[\epsilon^2\delta] & y_b[\delta] & \frac{m_{D_1}}{v_{heavy}}\epsilon^{-2}\delta & \frac{m_{D_2}}{v_{heavy}}\delta \end{pmatrix}, \tag{4.22}$$

where $\delta = \mathcal{O}(v_{light}/v_{heavy})$. The upper left-hand 3×3 -block corresponds to the SM quarks. There the diagonal entries are approximately the SM Yukawa couplings. Also small off-diagonal entries are produced. Each column is proportional to a quark mass. Therefore the entries above the diagonal are larger than the entries on the other side of the diagonal. The off-diagonal entries involving only SM quarks are suppressed by the ratio v_{light}/v_{heavy}. They can be made to satisfy the experimental bounds from the neutral meson mixing [254] by increasing the scale of the $SU(3)_L \times U(1)_X$-breaking. The entries involving quarks u, d, s, c and b are heavily constrained by the neutral meson mixing as it proceeds at tree-level. The top quark does not hadronize and the entries involving top quark are not constrained by neutral meson mixing at tree-level. Also the couplings involving exotic quarks are not constrained by the neutral meson mixing at tree-level. One has to go to 1-loop order to obtain bounds on the couplings involving top and the exotic quarks and as a result the bounds on them are very weak. The exotic quark masses are proportional to $v_{\rm heavy}$ and the entries depending on the masses of the exotic quarks do not vanish at high values of $SU(3)_L \times U(1)_X$ breaking. The lower bound on the masses of the exotic quarks is around 1 TeV [41].

The most stringent bound is coming from the B_d^0-$\bar{B}_d^0$-mixing. By assuming $v_{light} = \mathcal{O}(200\mathrm{GeV})$, this translates into a rough bound on the $SU(3)_L \times U(1)_X$-breaking scale:

$$v_{heavy} \gtrsim 5 \text{ TeV}. \tag{4.23}$$

It was numerically demonstrated that $SU(3)_L \times U(1)_X$ breaking scale around 5 TeV can satisfy the experimental constraints [214].

Chapter 5

Summary and outlook

The paper I studied the flavour violating effects originating from the flavon. The flavon inevitably possesses flavour violating couplings. The flavon can mix with the SM Higgs, providing the resulting 125 GeV mass eigenstate with flavour violating couplings. This was utilized in paper I to explain a CLFV Higgs decay $h \to \mu\tau$-signal. The $h \to \mu\tau$-signal has since gone away. The method is still useful. If flavour violating Higgs decay is truly observed in the quark or in the lepton sector, the Higgs-flavon mixing could provide an explanation. The CLFV decay signal of the Higgs might still come back with a much smaller branching ratio. The Higgs-flavon mixing could easily accomodate that, as t was already demonstrated to be able produce a large CLFV Higgs decay branching ratio, and still avoid the stringent CLFV bounds coming from decays $l_i \to l_j \gamma$ and $l_i \to l_j l_L l_l$, and $e - \mu$-conversion. The observation of the CLFV processes would be a clear sign of the physics beyond the Standard Model. The Froggatt-Nielsen mechanism could provide a source of the CLFV and at the same time explain the fermion mass hierarchy.

The papers II and III studied the embedding of the Froggatt-Nielsen mechanism into the 331-models. The paper II showed that the minimal scalar content of the 331-models with $\beta = \pm 1\sqrt{3}$ is capable of housing FN-mechanism. This is intriguing as the 331-models can also predict the number of fermion families. The FN331-model can thus economically explain the fermion mass hierarchy and the number of families. The traditional 331-models are plagued by the scalar mediated FCNCs of quarks at tree-level without a natural suppression mechanism. The FN-mechanism generates the structure of the quark mass matrices and allows one to study the suppression of the tree-level scalar mediated FCNCs of quarks as was done in paper III. The Higgs mediated quark FCNCs are suppressed by the ratio of the $SU(2)_L \times U(1)_Y$ and $SU(3)_X \times U(1)_X$-breaking scales. It was found that the breaking scale of $SU(3)_X \times U(1)_X$ as low as 5 TeV is enough to suppress the scalar mediated quark FCNCs.

The energy scale of the FN331-model is at least 5 TeV. The new gauge bosons and scalars are naturally of this mass scale. They are difficult to produce in the 13 TeV LHC, but maybe possible to see in the possible future colliders. The exotic quark masses acquire suppression from FN-mechanism and could be lighter than 5 TeV (but still at least 1 TeV) and one might get a signal from exotic quark production in the high luminosity phase of the LHC.

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